

29

TRANSACTIONS
OF THE
ROYAL SOCIETY OF SOUTH AFRICA

VOLUME VII.
1918-19.

WITH TWENTY-FOUR PLATES.

OHIO STATE
UNIVERSITY

CAPE TOWN:
PUBLISHED BY THE SOCIETY.

—
1919.

Q 85
C 23
V. 2

LONDON :

PRINTED BY ADLARD AND SON AND WEST NEWMAN, LTD.,
BARTHOLOMEW CLOSE, E.C. 1.

STAT2 OHIO
YTRSEVIRU

CONTENTS.

	PAGE
<i>Mestoma Antarcticum</i> (SP. NOV.) FROM BLOEMFONTEIN. By T. F. DREYER, B.A., Ph.D. (With text-figs. 1, 2)	1
COLOUR AND CHEMICAL CONSTITUTION: A STUDY OF THE PHTHALEINS AND RELATED COMPOUNDS. By JAMES MOIR, M.A., D.Sc., F.I.C.	5
NOTE ON AN EXPANSION OF THE PRODUCT OF TWO OBLONG ARRAYS. By SIR THOMAS MUIR, LL.D.	15
A SUMMARY OF THE DISTRIBUTION OF THE GENERA OF SOUTH AFRICAN FLOWERING PLANTS. (WITH SPECIAL REFERENCE TO THOSE FOUND IN THE DIVISIONS OF Uitenhage and Port Elizabeth.) By S. SCHÖNLAND.	19
NOTES ON (a) RADIATION PATTERNS OF THE TRANSFORMATION OF MAGNESIUM HYDROXIDE TO MAGNESIUM OXIDE. (b) SERPENTINE, MALACHITE, AND PSEUDOMORPH QUARTZ. (c) BULTFONTEIN APOPHYLLITE. (d) DIAMOND TESTS BY RADIATION PATTERNS. (e) DIFFRACTION FROM THE EDGES OF A SQUARE PLATE OF IODINE. By J. STEPH. V. D. LINGEN. (With Plates I—VI)	59
KIMBERLEY DIAMONDS: ESPECIALLY CLEAVAGE DIAMONDS. By J. R. SUTTON, M.A., Sc.D., F.R.S.S.A.	65
NOTE ON THE RESOLVABILITY OF THE MINORS OF A COMPOUND DETERMINANT. By SIR THOMAS MUIR, LL.D.	97
A LUNAR PERIOD IN THE RATES OF EVAPORATION AND RAINFALL? By J. R. SUTTON, Sc.D., Hon. Memb. R. Met. S.	103
COLOUR AND CHEMICAL CONSTITUTION: PART II.—THE SPECTRA OF THE MIXED PHTHALEINS AND OF THE SULPHONE-PHTHALEINS. By JAMES MOIR, M.A., D.Sc., F.I.C.	111
NOTE ON THE GENUS <i>Terfezia</i> ; A TRUFFLE FROM THE KALAHARI. By I. B. POLE EVANS, M.A., B.Sc., F.L.S. (With Plate VII)	117

	PAGE
NOTE ON THE ABNORMAL DEVELOPMENT OF THE GENITAL ORGANS OF <i>Jasus Lalandii</i> (MILNE EDW.). By W. VON BONDE, B.A. (With two text-figs.)	119
COLOUR AND CHEMICAL CONSTITUTION: PART III.—DERIVATIVES OF THE UNKNOWN ORTHO-PARA-PHENOLPHTHALEIN. By JAMES MOIR, M.A., D.Sc., F.I.C.	123
SPECTRUM-PHENOMENA IN THE CHROMIUM COMPOUNDS: BEING PART IV OF THE SPECTRUM OF THE RUBY AND EMERALD. By JAMES MOIR, M.A., D.Sc., F.I.C.	129
ON <i>Rana fuscigula</i> and <i>R. angolensis</i> . By G. A. BOULENGER, LL.D., D.Sc., F.R.S. (Published by permission of the Trustees of the British Museum)	131
HENRY HAROLD WELCH PEARSON, F.R.S.	139
THE TORSION PROBLEM FOR BODIES OF REVOLUTION. By E. T. STEGMANN. (Thesis approved for the Degree of Doctor of Science in the University of the Cape of Good Hope.) (With Plates A, B, VIII—XVIII)	147
NOTE ON THE STRUCTURE OF THE GENITAL ORGANS OF A TRUE HERMAPHRODITE. By H. V. EXNER, M.A. (From the Physiology Laboratory, South African College)	181
COLOUR AND CHEMICAL CONSTITUTION: PART IV.—THE REMAINING PHTHALEINS. By JAMES MOIR, M.A., D.Sc., F.I.C.	183
ON THE GENERA <i>Diplocystis</i> and <i>Broomeia</i> . By I. B. POLE EVANS, M.A., D.Sc., F.L.S., and AVERIL M. BOTTOMLEY, B.A. (With Plates XIX—XXIII)	189
SOUTH AFRICAN PERISPORIACEÆ. II.—REVISIONAL NOTES. By ETHEL M. DOIDGE, M.A., D.Sc., F.L.S. (With three text-figs.)	193
NOTE ON THE ADJUGATE OF BEZOUT'S ELIMINANT OF TWO BINARY QUANTICS. By Sir THOMAS MUIR, LL.D.	199
LUMINOSITY AND ITS ORIGIN IN A SOUTH AFRICAN EARTHWORM (<i>Chilota</i> sp.?). By J. D. F. GILCHRIST, M.A., Ph.D., D.Sc. (With Plate XXIV)	203

ROYAL SOCIETY OF SOUTH AFRICA.

LIST OF OFFICERS, FELLOWS, AND MEMBERS.

1917.

MEMBERS OF COUNCIL.

President.

L. PÉRINGUEY.

Vice-Presidents.

A. JASPER ANDERSON, G. S. CORSTORPHINE.

A. JASPER ANDERSON.
W. A. CALDECOTT.
G. S. CORSTORPHINE.
L. CRAWFORD, *Hon. Treasurer*.
A. L. DU TOIT.
S. S. HOUGH.

W. A. JOLLY, *Hon. Gen. Secretary*.
C. P. LOUNSBURY.
L. PÉRINGUEY.
W. F. PURCELL.
S. SCHÖNLAND.
B. DE ST. J. VAN DER RIET.

HONORARY FELLOWS.

Sir W. T. THISELTON DYER, *K.C.M.G., C.I.E., LL.D., Sc.D., F.R.S., F.L.S.*,
The Ferns, Witcombe, Gloucester.

Right Hon. Viscount MILNER, *G.C.B., G.C.M.G., P.C., M.A., LL.D., D.C.L.*,
Sturry Court, near Canterbury.

A. RAFFRAY, Curepipe, Mauritius.

FELLOWS.

Anderson, A. J., *M.A., M.B., D.P.H.*, City Hall, Cape Town.
Beattie, J. C., *D.Sc., F.R.S.E.*, The University of Cape Town, Cape Town.
Behr, H. C., 233, Broadway, New York.
Bohle, H., *M.I.E.E.*, The University of Cape Town, Cape Town.
Burt-Davy, J., *F.L.S., F.R.G.S.*, Burttholm, Vereeniging, Transvaal.
Caldecott, W. A., *B.A., D.Sc.*, P.O. Box 67, Johannesburg.
Corstorphine, G. S., *B.Sc., Ph.D.*, School of Mines and Technology, P.O. Box 1758, Johannesburg.

Crawford, L., *M.A., D.Sc., F.R.S.E.*, The University of Cape Town, Cape Town.
du Toit, A. L., *B.A., D.Sc.*, Geological Survey, Cape Town.
Franks, Sir Kendal, *Kt., C.B., B.A., M.D., F.R.C.S.I.*, P.O. Box 1858, Johannesburg.
Gilchrist, J. D. F., *M.A., D.Sc., F.L.S., Ph.D.*, The University of Cape Town, Cape Town.
Goetz, Rev. E., *S.J.*, The Observatory, Bulawayo, Southern Rhodesia.
Hall, A. L., *B.A., F.G.S.*, Geological Survey, P.O. Box 401, Pretoria.

- Halm, J. K. E., *Ph.D.*, Royal Observatory, Cape Town.
- Hewitt, J., *B.A.*, Albany Museum, Graham's Town, C. P., South Africa.
- Hough, S. S., *M.A., F.R.S.*, Royal Observatory, Cape Town.
- Innes, R. T. A., *F.R.A.S.*, Union Observatory, Johannesburg.
- Jolly, W. A., *D.Sc., M.B., Ch.B.*, The University of Cape Town, Cape Town.
- Juritz, C. F., *M.A., D.Sc., F.I.C.*, Agricultural Chemical Research Laboratory, Department of Agriculture, Cape Town.
- Kanthack, F. E., *M.I.C.E., M.I.M.E.*, Irrigation Department, Pretoria.
- Lehfeldt, R. A., *B.A., D.Sc.*, School of Mines and Technology, Johannesburg.
- Lounsbury, C. P., *B.Sc.*, Department of Agriculture, P.O. Box 513, Pretoria.
- Mally, C. W., *M.Sc.*, Pine Grove, Glebe Road, Rondebosch, Cape Town.
- Marloth, R., *Ph.D., M.A.*, Church Street, Cape Town.
- Mellor, E. T., *D.Sc.*, Central Mining and Investment Corporation, P.O. Box 149, Johannesburg.
- Moir, J., *M.A., D.Sc.*, Mining Chemist, Government Laboratory, P.O. Box 1080, Johannesburg.
- Morrison, J. T., *M.A., B.Sc., F.R.S.E.*, The University of Stellenbosch, Stellenbosch, South Africa.
- Moss, C. E., *M.A., D.Sc., F.L.S.*, School of Mines and Technology, Johannesburg.
- Muir, Sir Thos., *Kt., C.M.G., M.A., LL.D., F.R.S.*, Elmcote, Rondebosch, Cape Town.
- Nevill, E., *F.R.S.*, c/o High Commissioner's Office for Union of South Africa, Victoria Street, London.
- Ogg, A., *M.A., B.Sc., Ph.D.*, School of Mines and Technology, Johannesburg, South Africa.
- Péringuey, L., *D.Sc., F.E.S., F.Z.S.*, South African Museum, Cape Town.
- Pole-Evans, I. B., *M.A., B.Sc., F.L.S.*, Division of Botany, Department of Agriculture, P.O. Box 1294, Pretoria.
- Purcell, W. F., *B.A., Ph.D.*, Bergvliet, Diep River, Cape Prov., South Africa.
- Reunert, T., *M.I.C.E.*, P.O. Box 92, Johannesburg.
- Roberts, A. W., *D.Sc., F.R.A.S., E.R.S.E.*, Lovedale, Cape Prov., South Africa.
- Robertson, G. W., *M.R.C.S.*, Government Bacteriological Laboratory, Cape Town.
- Rogers, A. W., *Sc.D., F.G.S.*, Geological Survey, Heidelberg, Transvaal.
- Schönland, S., *M.A., Ph.D.*, Rhodes University College, Graham's Town, South Africa.
- Schunke-Hollway, H. C., *F.R.G.S.*, Belvedere, Wolmunster Park Road, Rosebank, Cape Town.
- Smith, Hon. Sir Charles Abercrombie, *Kt., M.A.*, Wynberg, Cape Prov., South Africa.
- Sutton, J. R., *Sc.D.*, P.O. Box 142, Kimberley, South Africa.
- Theiler, Sir Arnold, *K.C.M.G., D.Sc.*, Agricultural Department, P.O. Box 593, Pretoria.
- Van der Riet, B. de St. J., *Ph.D., M.A.*, The University of Stellenbosch, Stellenbosch, South Africa.
- Wagner, Percy A., *D.Ing., B.Sc., M.E.*, P.O. Box 1277, Pretoria.
- Warren, E., *D.Sc.*, The Government Museum, Maritzburg, Natal.
- Wilson, A. Marius, *M.D.*, Hof Street, Cape Town.
- Young, R. B., *M.A., D.Sc., F.R.S.E., F.G.S.*, School of Mines and Technology, Johannesburg.

MEMBERS.

- Alston, E. G., P.O. Spes Bona, Ceres, Cape Prov., South Africa.
- Alston, J. A., *M.R.C.S.*, Church Square, Cape Town.
- Andrews, W. H., *B.Sc., M.R.C.V.S.*, Department of Agriculture, P.O. Box 593, Pretoria.
- Arnold, G., *A.R.C.Sc., M.Sc., F.E.S.*, Rhodesia Museum, P.O. Box 240, Bulawayo, Southern Rhodesia.
- Barnard, K. H., *B.A.*, South African Museum, Cape Town.
- Battaerd, P., *M.D.*, The University of Cape Town, Cape Town.

- Baxter, W., *M.A.*, South African College School, Cape Town.
- Beare, G., *M.B.*, Kuruman, Cape Prov., South Africa.
- Beck, *Hon. Sir J. H. Meiring, Kt., M.D., M.R.C.P.Ed., F.R.S.E.*, The Drostdy, Tulbagh, Cape Prov., South Africa.
- Benjamin, L. E., *K.C., B.A., LL.B.*, Cape Times Buildings, Cape Town.
- Berry, *Sir W. Bisset, Kt., M.D.*, Ebdon Street, Queen's Town, South Africa.
- Bews, J. W., *M.A., D.Sc.*, Natal University College, Pietermaritzburg, Natal.
- Bleek, (Miss) D. F., Charlton House, Mowbray, Cape Town.
- Bolus, Mrs. Frank, *B.A., F.L.S.*, Sherwood, Kenilworth, Cape Town.
- Boyd, J., *F.I.C., A.R.T.C.*, P.O. Box 809, Johannesburg.
- Brain, Chas. K., *M.A., M.Sc.*, Division of Entomology, P.O. Box 513, Pretoria.
- Brauns, H., *M.D., Ph.D.*, Willowmore, Cape Prov., South Africa.
- Britstown Public Library, Britstown, South Africa.
- Brown, A., *M.A., B.Sc., F.R.S.E.*, The University of Cape Town, Cape Town.
- Buchanan, *Hon. Sir John, Kt.*, Claremont, Cape Town.
- Caink, T. G., Town House, King William's Town, Cape Prov., South Africa.
- Churchill, *Hon. Frank F.*, 69, Berea Park Road, Durban, Natal.
- Clark, G. M., *M.A., M.I.C.E.*, 15, Meischke's Buildings, Johannesburg.
- Clifford-Jones, E. G., *M.I.E.E., M.I.M.E.*, P.O. Box 231, Johannesburg.
- Cowper, Sydney, *C.M.G.*, Robinson Road, Wynberg, Cape Prov., South Africa.
- Dalton, J. P., *M.A., D.Sc.*, School of Mines and Technology, Johannesburg.
- Darter, G. B. Silver, *M.B., B.S.*, Victoria West, Cape Prov., South Africa.
- Davy, E. W., *F.R.H.S.*, Zomba, Nyasaland.
- de Fenton, J. *Ph.D.*, Seymour Library, Johannesburg.
- Dodd, J., O.F.S. Museum, Bloemfontein, O.F.S.
- Doidge, (Miss) E. M., *D.Sc., F.L.S.*, Division of Botany, P.O. Box 1294, Pretoria.
- Dreyer, T. F., *B.A., Ph.D.*, Grey University College, Bloemfontein, O.F.S.
- Dru Drury, E. G., *M.D., B.S., D.P.H.*, High Street, Graham's Town, South Africa.
- Dunstan, J. T., *M.D., B.S., M.R.C.S., L.R.C.P.*, Mental Hospital, Pretoria.
- Duthie, (Miss) A. V., *M.A.*, The University of Stellenbosch, Stellenbosch, South Africa.
- du Toit, P. J., Agricultural Department, Pretoria.
- Engelenburg, F. V., *Ph.D.*, P.O. Box 389, Pretoria.
- Evans, M. S., *C.M.G., F.Z.S.*, Durban, Natal.
- Exner, H. V., *M.A.*, Trinity College, Dublin.
- Eyles, F., *M.L.C.*, Department of Agriculture, Salisbury, Rhodesia.
- Fairbridge, W. G., *B.A.*, 141, Longmarket Street, Cape Town.
- Feltham, H. L. L., P.O. Box 46, Johannesburg.
- Figdor, J., *L.D.S.*, Central Chambers, Aliwal North, Cape Prov., South Africa.
- Fincham, G. H. H., *M.A.*, Hopewell, Imvani, Cape Prov., South Africa.
- Flint, *Rev. Wm., D.D.*, Wolmunster Park, Rosebank, Cape Town.
- Fourcade, H. G., Witte Els Bosch, District Humansdorp, Cape Prov., South Africa.
- Fraumes, P. Ross, P.O. Box 148, Johannesburg.
- Friedlander, Adv. C., *F.R.G.S.*, Marais Court Chambers, Wale Street, Cape Town.
- Galpin, E. E., *F.L.S.*, Bank of Africa, Queen's Town, South Africa.
- Gericke, *Capt. O. M., S.A.M.C., L.R.C.P., L.R.C.S., M.B., Ch.B.*, No. 1 General Hospital, Wynberg, South Africa.
- Gorges, E. H. L., Administrator, Windhuk, South-West Africa.
- Grimmer, J. R., De Beers Consolidated Mines, Ltd., Kimberley.

- Grobbelaar, C. S., *M.A.*, Heemsted, Riebeeck Street, Stellenbosch, Cape Prov., South Africa.
- Hamlin, E. J., *B.Sc.*, Electric Light Works, Stellenbosch, Cape Prov., South Africa.
- Hardenberg, C. B., *M.A.*, Department of Agriculture, New Hanover Rail, Natal.
- Harger, H. S., P.O. Box 1945, Johannesburg.
- Haughton, Sidney H., *B.A.*, South African Museum, Cape Town.
- Hayward, S. H., *L.D.S.*, Colonial Mutual Buildings, Cape Town.
- Henkel, J. S., Forest Department, Pietermaritzburg, Natal.
- Herman, C. L., *M.B.*, *C.M.*, *M.R.C.S.*, Mount Nelson Hotel, Cape Town.
- Heymans, A., *M.D.*, P.O. Box 259, Pretoria.
- Holtzhausen, D., *B.A.*, Irrigation Department, Pretoria, Transvaal.
- Howard, R. N., *M.R.C.S.*, O'okiep, Cape Province, South Africa.
- Hugo, D. de V., *M.B.*, *C.M.*, Mansion House Chambers, Cape Town.
- Humphreys, W. A., *B.A.*, Ohlange Institute, Phoenix, Natal.
- Hutcheon, James, *M.A.*, *F.R.G.S.*, School of Mines and Technology, Johannesburg.
- Jacot, E., *B.A.*, The University of Cape Town, Cape Town.
- Jagger, J. W., *F.S.S.*, *M.L.A.*, St. George's Street, Cape Town.
- Janisch, N., Blinkbonnie, Camp Ground Road, Newlands, Cape Town.
- Janse, A. J. T., Normal College, P.O. Box 855, Pretoria.
- Jardine, Major W., P.O. Box 153, Cape Town.
- Johnston, W., Rhodes Buildings, Cape Town.
- Kehoe, D., *M.R.C.V.S.*, Veterinary Research Laboratory, P.O. Box 593, Pretoria.
- Kent, Thomas P., *M.A.*, The University of Cape Town, Cape Town.
- Ketchen, A. D., *M.D.*, Clonbrook, 82, Orange Street, Cape Town.
- Kolbe, Rev. F. C., *B.A.*, *D.D.*, St. Mary's Presbytery, Cape Town.
- Krapohl, J. H. C., *B.A.*, Concordia, Cape Prov., South Africa.
- Lawrence, F. J., *C.C.* & *R.M.*, Tulbagh, Cape Prov., South Africa.
- Legat, C. E., Department of Forestry, Pretoria.
- Le Roux, T. H., *B.A.*, *Litt.D.*, University College, Pretoria.
- Leslie, C. D., P.O. Box 1167, Johannesburg.
- Leslie, T. N., P.O. Box 23, Vereeniging, Transvaal.
- Lester, J. B., *M.D.*, 80, Queen Victoria Street, Cape Town.
- Leviser, M., P.O. Box 253, Bloemfontein, O.F.S.
- Lewis, Alfred D., *M.A.*, Irrigation Department, Bloemfontein, O.F.S., South Africa.
- Lithman, K. V., P.O. Box 640, Cape Town.
- Lloyd, A. C. G., *B.A.*, South African Public Library, Cape Town.
- Logeman, W. H., *M.A.*, Grey University College, Bloemfontein, O.F.S.
- Lotz, H., *Ph.D.*, Berliner Strasse 57, Charlottenburg, Germany.
- Lyle, J., *M.A.*, Grey College School, Bloemfontein, O.F.S.
- Malan, D. E., *M.A.*, c/o Mrs. F. E. Malan, Murray Street, Wellington, Cape Prov., South Africa.
- Malleson, P. R., Ida's Valley, Stellenbosch, Cape Prov., South Africa.
- Mansergh, C. W. L., *I.S.O.*, Provincial Administration, 3, Wale Street, Cape Town.
- Marshall, G. A. K., *D.Sc.*, *F.E.S.*, *F.Z.S.*, 6, Chester Place, London, W. 2.
- Menmuir, R. W., *M.Inst.C.E.*, Church Square, Cape Town.
- Methuen, Hon. Paul A., *F.Z.S.*, 349, Jorissen Street, Pretoria.
- Mitchell, D. T., *M.R.C.V.S.*, Department of Agriculture, Research Laboratory, Pietermaritzburg, Natal.
- Moffat, J. B., Senior R. M., Transkei, South Africa.
- Molengraaf, G. A. F., *Ph.D.*, Voorstraat 60, Delft, Holland.
- Nicholson, G. T., *M.Inst.C.E.*, Resident Engineer, Alfred Docks, Cape Town.

- Nobbs, E., *Ph.D., B.Sc.*, Department of Agriculture, Salisbury, Rhodesia.
- Orpen, J. M., Mon Asile, 43, St. Mark's Road, East London, Cape Prov., South Africa.
- Park, Sir Maitland H., Kt., *M.A., LL.D.*, Cape Times Buildings, Cape Town.
- Pealing, G. H., *M.Sc.*, The University of Cape Town, Cape Town.
- Petersen, Julius, *M.B., C.M.*, Hope Street, Cape Town.
- Petty, F. W., Agricultural College, Elsenberg, Mulder's Vlei, Cape Prov., South Africa.
- Pickstone, H. E. V., Groot Drakenstein, Paarl, Cape Prov., South Africa.
- Poch, R., *M.D.*, Maximilian Platz, Vienna.
- Potts, G., *M.Sc., Ph.D.*, 91, Park Road, Bloemfontein, O.F.S.
- Price, Bernard, *M.I.E.E., A.M.Inst.C.E.*, P.O. Box 2671, Johannesburg.
- Ray, Robt., *B.Sc.*, The University of Cape Town, Cape Town.
- Rindl, M., *D.Ing.*, Grey University College, Bloemfontein, O.F.S.
- Ritchie, W., *M.A.*, The University of Cape Town, Cape Town.
- Roberts, Rev. N., The Vicarage, Orchards, Johannesburg.
- Rose, J. G., Union Chemical Laboratory, Cape Town.
- Rosevears, W. N., *M.A.*, Natal University College, Maritzburg, Natal.
- Russell, W. A., *M.A.*, Education Department, Cape Town.
- Sandes, P. Lindsay, *M.A., M.D.*, S. A. Mutual Buildings, Cape Town.
- Schreiner, Hon. W. P., *C.M.G., K.C., M.A., LL.M.*, High Commissioner's Office for the Union of South Africa, Victoria Street, London, S.W.
- Scully, Gerald C., *M.A.*, c/o W. C. Scully, 15, Reservoir Road, Queenstown, Cape Prov., South Africa.
- Sharples, Rev. William G., Vicar of Formosa, Plettenberg Bay, Cape Prov., South Africa.
- Silberbauer, C. F., 14, Keerom Street, Cape Town.
- Simons, L., *B.Sc.*, The University of Cape Town, Cape Town.
- Simpson-Wells, A., *M.A., M.D., F.R.C.S.E.*, 26, Kloof Road, Cape Town.
- Skaife, S. H., *B.A.*, Government Experimental Station, Rose Bank, Cape Town.
- Smith, F. B., *B.A.*, Secretary for Agriculture, Union of South Africa, Pretoria.
- Smith, Hugh, *M.D.*, Derwent House, Hof Street, Cape Town.
- Smuts, Rt. Hon. J. C., *B.A., LL.B.*, Department of Defence, Union of South Africa, Pretoria.
- Snape, A. E., *M.Sc., A.M.Inst.C.E.*, The University of Cape Town, Cape Town.
- Snyman, Mrs. Lily C., Klipfontein, Postmasburg, Griqualand West, South Africa.
- Stegmann, E. T., *B.A., D.Sc.*, The University of Stellenbosch, Stellenbosch, South Africa.
- Stephens, (Miss) E. L., *B.A., F.L.S.*, The University of Cape Town, Cape Town.
- Stewart, C. M., *B.Sc.*, Meteorological Department, P.O. Box 399, Pretoria.
- Stewart, T., *F.G.S., M.I.C.E.*, St. George's Street, Cape Town.
- Stott, C. H., *F.G.S.*, P.O. Box 7, Pietermaritzburg, Natal.
- Swierstra, C. J., Transvaal Museum, Pretoria.
- Taylor, W. H., Public School, Mowbray, Cape Town, South Africa.
- Thomson, R. B., *M.B., Ch.B., F.R.S.E.*, The University of Cape Town, Cape Town.
- Tooke, W. Hammond, Kendal Lodge, Graham's Town, South Africa.
- Transvaal Museum, P.O. Box 413, Pretoria.
- Tredgold, Hon. C. H., *LL.B., K.C.*, Attorney-General's Office, Salisbury, Rhodesia.
- Tregarthen, W. Coulson, *Mus.B.*, The Hermitage, Queen's Town, South Africa.
- Tucker, Hon. W. K., *C.M.G.*, 65, Pretoria Street, Hospital Hill, Johannesburg.
- Van der Bijl, P. A., *D.Sc., F.L.S.*, Natal Herbarium, St. Thomas Road, Berea, Durban, Natal.

- Van der Lingen, J. Steph., *B.A.*, The University of Cape Town, Cape Town.
- Versfeld, J. J., *L.R.C.S., L.R.C.P.*, Stellenbosch, Cape Prov., South Africa.
- Von Bonde, W. K., *B.A.*, Grey University College, Bloemfontein, O.F.S.
- Wager, H. A., *A.R.C.S.*, Transvaal University College, Pretoria.
- Walker, A. R. E., *B.Sc., A.R.C.S.*, The University of Cape Town, Cape Town.
- Walker, James, *M.R.C.V.S.*, Veterinary Research Laboratory, P.O. Box 593, Pretoria.
- Wallis, A. H., 3, First Avenue, Mafeking, Cape Prov., South Africa.
- Walsh, A., P.O. Box 39, Cape Town.
- Walsh, Lionel H., Kenilworth, Cape Town.
- Walters, J. A. T., *B.A.*, Department of Agriculture, Salisbury, Rhodesia.
- Webber, W. S., P.O. Box 1088, Johannesburg.
- Whaits, Rev. J. H., The Rectory, Graaff Reinet, Cape Prov., South Africa.
- Williams, D., *B.Sc.*, Rhodes University College, Graham's Town, South Africa.
- Wilman, (Miss) M., The Museum, Kimberley.
- Wood, D. J., *M.B., C.M.*, Church Square, Cape Town.
- Wood, H. E., *M.Sc., F.R.A.S.*, Union Observatory, Johannesburg.
- Young, A., *M.A., D.Sc.*, The University of Cape Town, Cape Town.
- Zealley, A. E. V., *A.R.C.S.*, Geological Survey, Bulawayo, Southern Rhodesia.

TRANSACTIONS
OF THE
ROYAL SOCIETY OF SOUTH AFRICA.
VOL. VII.

MESTOMA ANTARCTICUM (SP. NOV.) FROM BLOEMFONTEIN.

By T. F. DREYER, B.A., PH.D.

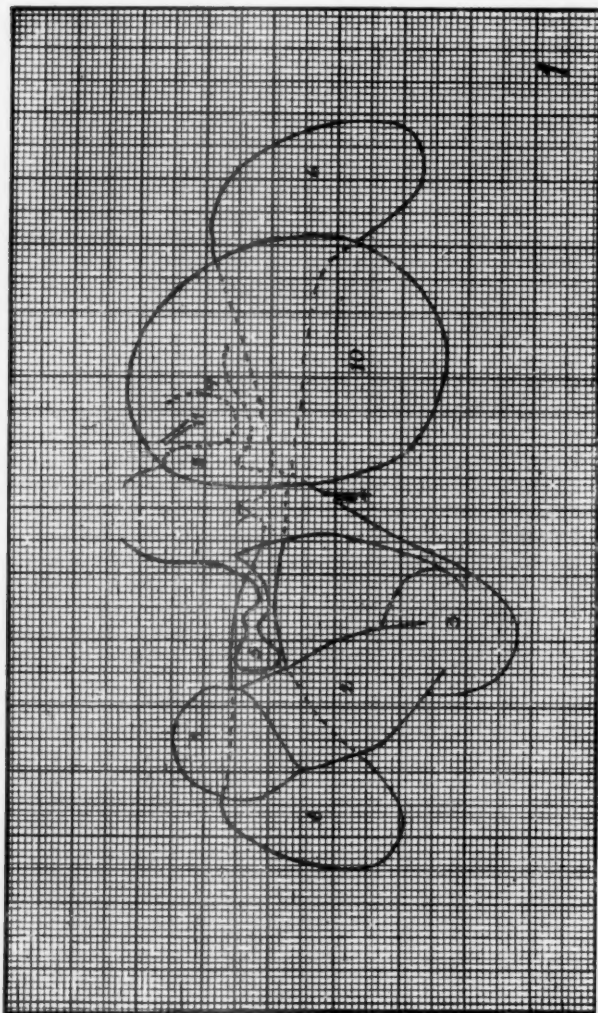
Locality.—A small pond on clay soil near Bloemfontein.

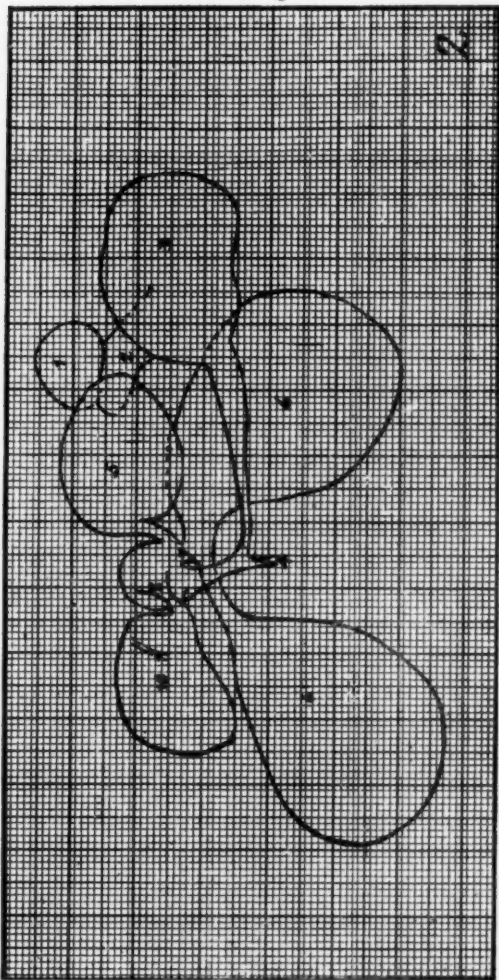
Size.—A very slender worm, tapering towards both ends, and, when fully extended, about 7 mm. in length and about 1 mm. broad. Five specimens were collected in November, and none of them contained eggs; but after having been kept in the laboratory for two days two of them had a brick-red winter egg in each uterus.

Description.—The anterior end can be withdrawn by special bands of muscle fibres, but it is not sharply set off from the rest of the body. The epithelium is not pigmented, the faintly yellowish-brown pigment being restricted to the mesenchyme. In the preserved specimens the body is roughly square in cross-section, the dorso-lateral ridges being, however, more distinct than the ventro-lateral ones. The two uteri simple, backwardly directed sacs. The mouth and the genital aperture open together. The pre-pharyngeal gut is appreciably shorter than the postpharyngeal portion.

General.—From the above it will be seen that the present species is very similar to, one may almost say identical with, *M. mutabile* from Tierra del Fuego. This similarity is also shown in the structure of the reproductive organs. The testes are united posteriorly and also just in front of the pharynx; each testis is a sacculated tube and is divided into an anterior and a posterior portion, since the tube contains no germ-plasm in the region immediately over the mouth. The two vasa deferentia are united into a short seminal duct. The only difference between the South African and the South American species seems to be in the structure of the oviduct and its receptaculum seminis. Böhmig (see v. Graff, 'Turbellaria in Das Thierreich') figures a long, thin oviduct with a laterally situated receptaculum, which is

FIGS. 1 AND 2.—RECONSTRUCTION OF THE GENITAL ORGANS OF *M. antarcticum*—
(1) BEFORE; (2) AFTER EGGS HAVE BEEN FORMED.





1. Ovary. 2. Oviduct. 3. Receptaculum seminis (empty in Fig. 1). 4. Yolk-duct.
5. Bursa copulatrix (empty in Fig. 1). 6. Uterus (empty in Fig. 1). 7. Uterine opening into atrium. 8. Penis. 9. Opening of ductus ejaculatorius into the vesicula. 10. Vesicula seminalis. 11. Opening of ductus seminalis into the vesicula.]

very unlike my reconstructions. Luther ('Z. wiss. Zoo.,' v. 77, p. 235), however, is of opinion that Böhmig's lateral receptaculum is an artefact. Even if this is so, the relative thickness of the ovary and oviduct still presents a difference which is striking when it is borne in mind that the oviduct is not a hollow organ and therefore not capable of much expansion. *M. mutabile* is also described as possessing a bursa copulatrix, which is very considerably larger than the vesicula seminalis; a comparison of my Figs. 1 and 2 will, however, show that the size of these organs differs according to the amount of seminal fluid which they contain.

Conclusion.—*M. antarcticum* shows an almost negligible amount of variation from *M. mutabile* from Tierra del Fuego; whether this similarity is due to parallel evolution or to a former land connection or to dispersal by birds must remain an open question, although it is noteworthy that the nearest approximation to these two species is *M. canum* from S.W. Australia.

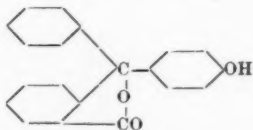
COLOUR AND CHEMICAL CONSTITUTION: A STUDY OF THE PHTHALEINS AND RELATED COMPOUNDS.

By JAMES MOIR, M.A., D.Sc., F.I.C.

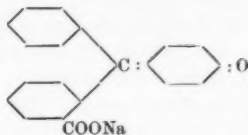
Whilst it has been known for a long time that if a coloured organic substance or dye is converted into a substitution product (*e.g.* aurine into eupittonic acid), the colour is much deepened, yet there are remarkably few systematic researches on record which give any clue as to the factors governing this phenomenon, important as it is from the practical point of view. Of the recorded work practically all belongs to the azo-dye series—for example, J. T. Hewitt's investigation into the changes of colour produced in nitrobenzeneazo-*a*-naphthol by adding the sulphonic group to its molecule in the different possible positions; also the recent interesting and systematic work emanating from the Dacca College, India.

The present paper purports to be, as far as possible, a complete study of the phthaleins and of other derivatives of the parent substance fuchsone, and the work has consisted, briefly, in an examination of the colour and absorption-spectrum of every phthalein and oxy-derivative of triphenylcarbinol that could be made with the materials at my disposal, and of an attempt to include all the numerous and puzzling data in a comprehensive theory of colour-change.

The simplest phthalein possible is monoxydiphenylphthalide (also known as phenylphenolphthalein, and in Germany incorrectly as "benzolphenolphthalein"), a colourless substance usually depicted by the formula

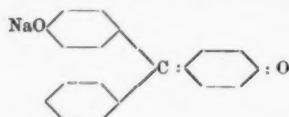


which, when dissolved in alkali, becomes coloured (a rather purplish pink), and is then assumed to have been converted into the quinonoid form:

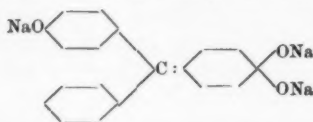


As will be seen from the latter formula, the substance is the ortho-carboxylic acid of fuchsone, the latter being the inner-anhydride of para-oxy-triphenylcarbinol, and its systematic name being diphenyl-methylene-quinone ("diphenyl-chinomethan" in Germany). The centre of its absorption band lies at about $\lambda 560$ (frequency 17.86). Fuchsone itself cannot be investigated by this method, since it is a neutral body giving no colour in alkali, though itself coloured (orange).

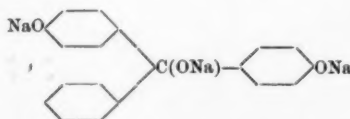
Another related compound of simple constitution may be taken next, although it is not a phthalein—viz. benzaurine or 4'-oxyfuchsone



This differs chemically from the preceding compound in having no ortho- CO_2Na group, but has a para- ONa group instead (both phenyl groups of the fuchsone molecule having the same function). This alkaline group confers solubility in water, with colour, as in the foregoing case, but the colour is now a reddish-purple with absorption-band at $\lambda 570$ (frequency 17.55). The addition of strong NaOH to benzaurine first deepens the colour to purple (absorption-band $\lambda 585$), probably due to the formation of



and, on warming, bleaches the colour owing to the destruction of the quinonoid ring to form



or the trisodium salt of dioxytriphenylcarbinol, by migration of an ONa group from the end to the middle of the system. The same behaviour is shown by monoxydiphenylphthalide in an exaggerated degree, only a slight alkalinity sufficing to bleach it.

The effect of the addition of a second para- OH (or ONa) group to the molecule may next be studied. Ordinary phenolphthalein is the para- OH derivative of monoxydiphenylphthalide, and ordinary aurine is the corresponding OH derivative of benzaurine. The effect is surprising, since in both cases the absorption-band moves down the spectrum into the green,

phenolphthalein having its band at $\lambda 554$ (frequency 18.05), and aurine having its band at $\lambda 530$ (frequency 18.88). In the former case the change caused by the extra $-\text{OH}$ group is insignificant, the frequency having only risen by 0.2; but in the case of aurine the difference is very great, the frequency being raised by 1.6 or nearly 10 per cent. This abnormal behaviour of aurine is doubtless connected with its almost perfect chemical symmetry as compared with all the other compounds which I deal with: as anhydride of $\text{C}(\text{OH})(\text{C}_6\text{H}_4\text{OH})_3$, it has always three paths by which it may become quinonoid in the form $\text{C}(\text{C}_6\text{H}_4\text{O})_3\text{H}_2$. It may be noted that concentrated alkali raises the band of aurine to $\lambda 539$.

Again, phenolphthalein may be looked on as the *o*-carboxylic acid of benzaurine, whence we infer that this $-\text{CO}_2\text{H}$ group causes a change of frequency of 18.05–17.55 or 0.5, from which the theoretical frequency of the parent substance fuchsone can be deduced to be about 17.3—viz. that of monoxydiphenylphthalide less 0.5. Thus, the passage from fuchsone to oxy-fuchsone (benzaurine), with destruction of the symmetry, involves a rise of only 0.2 in the frequency, whereas the entrance of a second OH group, giving aurine and restoring the symmetry, leads to a rise of 1.6 in the frequency. This theoretical figure for alkaline fuchsone is confirmed by the fact that it gives a frequency of 22.0 when dissolved in H_2SO_4 ; this is 1.4 above the value for monoxydiphenylphthalide, and benzaurine is found to give a frequency in H_2SO_4 , which is 1.5 above that of phenolphthalein.

To complete the series I have prepared a small quantity of the unknown aurine-*o*-carboxylic acid (which is also the 4'-oxy-derivative of phenolphthalein) by condensing 4'-oxyphthalic acid with phenol. It was found to give a pink solution in caustic alkali, possessing a band at $\lambda 542$, exactly half-way between those of aurine and phenolphthalein.

STUDY OF THE SUBSTITUTED PHENOLPHTHALEINS.

These were chosen for investigation on account of their easy synthesis rendering it possible to make a large number of derivatives; but it was also found that their absorption-bands are remarkably definite as a rule, thus rendering accurate measurements of their centres possible, which is generally not the case with the commoner dye-stuffs, particularly those of the azo-class which have been hitherto investigated. The centre of the band can generally be estimated within one unit of wave-length (in 10^{-9} metres) in the green, but with less accuracy at the red end; consequently the frequency values in Tables I and II can, as a rule, be trusted to within about 0.05 unit—i. e. to an extent considerably finer than that to which the eye can appreciate differences of shade of colour.

A set of 50 phthaleins has been prepared and examined, and the results are classified in two tables. Table I gives the results for the simple deriva-

TABLE I.
Colours and Spectra of Derivatives of Phenolphthalein in Alkaline Solution.


No.	Name.	Source.	Alkaline Colour.	Centre of Absorption Band $\lambda \times 10^3$ cm.	Frequency $\frac{1}{\lambda} \times 10^3$ per cm.	Diminution of Frequency.	Cause of Diminution.
1	Phenolphthalein	PA and phenol	Pink	554	1805	—	—
2	Monomethylether of phenolphthalein	ABA and phenol	"	555	1800	0.05	One $p\text{-CH}_3$.
3	Methylallylatephenolphthalein	PA and wintergreen oil.	"	557	17.95	0.10	Two $o\text{-CO}_2\text{CH}_3$.
4	Phenol-3-oxyphtalein	3-oxyphtalic acid and phenol	"	557	17.95	0.10	One OH in phthalic ring.
5	Phenolphthalein - disulphonic acid	PA and phenol- o -sulphonic acid	"	557	17.95	0.10	Two $o\text{-SO}_3\text{Na}$.
6	Phenolphthalein - o -dicarboxylic acid	PA and salicylic acid	"	559	17.90	0.15	Two $o\text{-CO}_2\text{Na}$.
7	Phenolphthalein - m -dicarboxylic acid	PA and m -oxybenzoic acid	"	559	17.90	0.15	Two $m\text{-CO}_2\text{Na}$.
8	Phenolphenyphthalein	BBA and phenol	"	560	17.85	0.20	Absence of one $p\text{-OH}$.
9	O -cresolphenyphthalein	BBA and o -cresol	Purplish-pink	564	17.75	0.30	Do., with gain of one $o\text{-CH}_3$.
10	Phenoltetrabromophthalein	TBP and phenol	"	565	17.70	0.35	Four Br in phthalic ring.
11	Dibromophenolphenyphthalein	Bromination of No. 8	"	568	17.60	0.45	Two $o\text{-Br}$ minus one $p\text{-OH}$.
12	O -cresolphthalein	PA and o -cresol	"	571	17.50	0.55	Two $o\text{-CH}_3$.
13	O -dibromo- o -cresolphthalein	Bromination of No. 12	Purple	578	17.30	0.75	Two $o\text{-CH}_3$ plus two $o\text{-Br}$.
14	Tetrabromophenolphthalein	Bromination of No. 1	"	583	17.15	0.90	Four Br (ortho) in phenol rings.
15	Tetridophenolphthalein	Iodination of No. 1	Bluish-purple	586	17.05	1.00	Four I (ortho) in phenol rings.
16	Catecholphthalein (NaHCO ₃)	PA and pyrocatechol	"	586	17.05	1.00	Two $o\text{-OH}$ (not ONa).
17	M -cresolphthalein	PA and metacresol	"	590	16.95	1.10	Two $m\text{-CH}_3$.
18	Thymolphthalein	PA and thymol	Blue	597	16.75	1.30	Two $m\text{-CH}_3$ plus two $o\text{-C}_2\text{H}_5$.
19	M -chlorophenolphthalein	PA and m -chlorophenol	"	597	16.75	1.30	Two $m\text{-Cl}$.

20	Guaiacolphthalein	PA and guaiacol	Blue	599	16.70	1.35	Two α -OCH ₃ .
21	Carvacrophthalein	PA and carvacrol	"	601	16.65	1.40	Two α -CH ₃ plus two m -C ₂ H ₅ .
22	Tetrahydronaphtholphthalein 2.3 (butylene-naphthol-phthalein)	PA and tetrahydro- α -naphthol	Dichroic	602	16.90	1.45	Two $-(CH_2)_4$ joined o and m .
23	Octobromophenolphthalein	Bromination of No. 10	Blue	608	16.45	1.00	Four Br in phthalic ring and two Br in each phenol ring (ortho).
24	Protocatechuiphthalein	PA and protocatechuic acid	"	612	16.35	1.70	Two α -OH plus two m -CO ₂ Na.
25	Thymoltetrabromophthalein	TBP and thymol	Greenish-blue	623	16.05	2.00	Four Br in phthalic ring, and two m -CH ₃ plus two α -C ₂ H ₅ .
26	Dibromocarvacrophthalein	Bromination of No. 21	"	623	16.05	2.00	Two α -Br, two α -CH ₃ , and two m -C ₂ H ₅ .
27	Dibromothymolphthalein	Bromination of No. 18	"	625	16.00	2.05	Two α -Br, two m -CH ₃ , and two α -C ₂ H ₅ .
28	O-diaminophenolphthalein	Reduction of dinitro-compound	"	630	15.90	2.15	Two α -NH ₂ .
29	Catecholphthalein (NaOH)	As No. 16	Green	632	15.35	2.70	Two α -ONa (or orthoquinoid linkage).
30	α -naphtholphthalein	PA and α -naphthol	"	632	15.10	2.95	Two $-\text{CH}:\text{CH}:\text{CH}:\text{CH}-$.
31	α -naphthol-3-oxyphtalein	From 3-oxyphtaleic acid	"	662	15.10	2.95	No apparent change for this OH.
32	Dibromoguaiacolphthalein	Bromination of No. 20	Dichroic	673	14.85	3.2	Abnormally high.
33	α -naphtholtetrabromophthalein	TBP and naphthol	Yellow-green	678	14.75	3.3	Naphthalene rings and four Br.
34	α -naphtholphenylphthalein	BBA and naphthol	Greenish-yellow	About 720	13.9 (?)	4.2 (?)	One naphthalene ring; absence of one OH.
35	Dibromocatecholphtalein	Bromination of No. 16	Dirty purple	About 720	13.9 (?)	4.2 (?)	(Second band λ 489.)
36	o -dinitrophenolphthalein	Nitration of No. 1	Yellow	Off the	visible	spectrum	Two α -NO ₂ .
37	m -dinitrophenolphthalein	PA and m -nitrophenol	Brown	"	"	"	Two m -NO ₂ .
38	Resorcinolphenylphthalein	BBA and resorcinol	Orange	"	"	"	Remarkable isomer of phenolphthalein.
39	Aurine carboxylic acid	4-oxyphtaleic acid and phenol	Pink	542	18.45	- 0.4	Increase instead of diminution.

PA = phthalic anhydride. TBP = tetrabromophthalic anhydride. BBA = α -benzoylbenzoic acid. ABA = anisoylbenzoic acid.

TABLE II.

Spectra of Derivatives of Fluoresceine (2-2'-ozo-phenolphthalein).

No.	Name.	Source.	Centre of Absorption Band λ .	Frequency.	Diminution of Frequency.	Cause of Diminution.
1	Fluoresceine	PA and resorcinol	499	20.05	—	—
2	3-oxyfluoresceine	3-oxy PA and resorcinol	489	20.45	-0.40	Increase due to 3-oxy group in phthalic ring.
3	Quinolphthalein	PA and quinol	493	20.25	-0.20	Increase due to moving of two OH from <i>p</i> to <i>m</i> .
4	Phloroglucin-phthalein	PA and phloroglucinol	497	20.15	-0.10	Constitution unknown.
5	Resorcinbenzeine	Benzoic acid and resorcinol	501	19.95	0.10	Loss of phthalic -CO ₂ Na.
6	Iso-eosine	TBP and resorcinol	503	19.90	0.15	Addition of four Br in phthalic ring.
7	Succinyl resorceine	Succinic acid and resorcinol	504	19.85	0.20	Replacement of  CO ₂ H by -CH ₂ CH ₂ CO ₂ H.
8	Eosine (tetrabromofluoresceine)	—	516	19.35	0.70	4 Br (ortho) in phenol rings.
9	Erythrosine (tetriodofluoresceine)	—	525	19.05	1.00	4 I (ortho) in phenol rings.
10	Galleine (pyrogallolphthalein)	—	About 543	—	—	? Four -OH groups.
11	Diethylrhodamine (neutral)	—	553	18.10	1.95	Two -OH replaced by two NEt ₃ groups.
12	Diethylrhodamine in acid	—	540 and 559	17.90 and 18.50	1.55 and 2.15	Replacement by NH ₄ ⁺ ion and by NH ₄ Cl.
13	Galleine (NaOH)	—	560 broad	17.85	About 2.2	? Four -ONa groups.

tives of phenolphthalein, and Table II gives those which possess the fluorane-oxygen linkage. As will be seen from column 4, the alkaline colours of these phthaleins cover every possible shade due to absorption of light, from the centre of the spectrum to the infra-red, which, from the practical point of view, is very important, since the order of efficiency of the different substituents is probably the same for many other classes of dyes, from which the colour of many unknown dyes could be predicted; at all

events, a similar order appears to reign among the other classes of dyes which have been similarly investigated.

The following are some of the deductions from these observations:

(1) The weight of the substituting material is of little importance compared with its volume; thus the addition of 8 bromine atoms has not much more effect than the addition of 2 *o*-methoxyl groups, and 4 iodine atoms have the same effect as 2 hydroxyl groups. This agrees with the usually accepted view that the atomic volumes of bromine and iodine are not much greater than that of hydrogen, whilst those of carbon, nitrogen, and oxygen are larger.

(2) The position of the substituting group, whether ortho- or meta- to the phenolic hydroxyls, is important; thus the meta-position has more than twice the effect of the ortho-position; in addition, it may be noted that groups actually attached to a phenolic hydroxyl have very little effect.

(3) The two first conclusions may be put in one by supposing that the shifting effect of substituting groups on the absorption-band of phenolphthalein is essentially due to mutual interference of the two phenol rings; such groups as project much from the ring, or are nearer to the junction (the central carbon atom), cause a great effect—an effect which is heightened when the group is unsaturated as in naphtholphthalein, the explanation being that the unsaturated ring is more rigid and causes greater interference to rotation than any combination of simply attached groups could do.

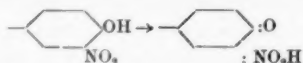
(4) Groups in the phthalic ring have comparatively little effect, which is also the case with acidic groups in the phenolic rings.

(5) Methoxyl groups have a remarkably large effect. This suggests that the oxygen atom is comparatively large in volume, and seems to support the notion suggested by the author in 1906 that oxygen is CX_2 where X is the unknown element (? halogen) of atomic weight 2.

(6) The differences for the same substituent are not always similar—for example, in the different bromo-derivatives—a fact which seems to suggest that there is a mutual action between the groups themselves tending to distort the quinonoid ring which is the cause of the colour. Thus it is striking that the isomeric dibromothymol-, and dibromocarvacrol-phthaleins have almost the same spectrum, whereas there is a slight difference between thymol- and carvacrol-phthaleins lying in the opposite direction.

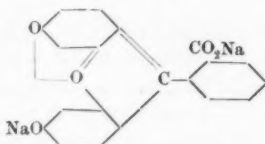
(7) The effect of the bridge oxygen in fluorescein and its derivatives is profound, and must be ascribed to the prevention of rotation of the phenol rings. Contrary to expectation, however, the frequency of the absorption-band is raised—viz. from 18.05 to 20.05—in passing from phenolphthalein to fluorescein, whereas the usual effect of mere loading of a molecule is to lower the frequency, as seen in the 39 derivatives of phenolphthalein described in Table I. It is probable, therefore, that the case of fluorescein is similar to that of the exception in the phenolphthalein series—viz. its

nitro-compound—in that the 2:2'-oxo group distinguishing fluorescein from phenolphthalein must take part in a quinonoid change which totally alters the configuration of the molecule, just as the quinonoid change

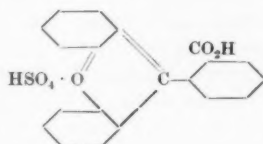


has completely altered the nature of dinitrophenolphthalein.

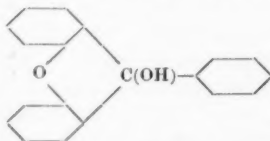
Now, the only orthoquinonoid configuration which can be planned for fluorescein is



in which a phenolic acid group has combined with a basic oxonium group. It should be noted that alternative methods of making this linkage are excluded by the fact that resorcin-benzeine, which is fluorescein minus the CO_2H group, shows the same behaviour as fluorescein, and that fluorane in strong H_2SO_4 shows a similar colour and fluorescence which, I suggest, is due to the linkage



If the still simpler substance of the annexed constitution



could be prepared, its behaviour in H_2SO_4 would, to some extent, settle the question of the cause of the colour and fluorescence of this class of substances.

PHTHALEINS DISSOLVED IN CONCENTRATED SULPHURIC ACID.

The author has discovered that the phthaleins and aurines when dissolved in H_2SO_4 all give coloured solutions with definite absorption-bands

TABLE III.
Spectra and Colours of Phthaleins in conc. H_2SO_4 Solution.

A. Phthaleins: constant 4.55.

No.	Name.	Frequency in Alkali = x .	Frequency in H_2SO_4 observed = y .	Colour.	Calculated Value of y .
1	Phenolphthalein	18.05	20.20	Salmon-yellow	20.25
2	Phenolphthalein methyl ether	18.00	20.05	"	20.18
3	4-oxyphenolphthalein	18.45	20.80	"	20.85
4	Phenoltetrabromophthalein	17.70	19.95	"	19.73
5	O-cresolphthalein	17.50	19.65	Pink	19.43
6	Dibromo-o-cresolphthalein	17.30	18.85	"	19.13
7	Tetrabromophenolphthalein	17.15	18.90	"	18.90
8	Thymolphthalein	16.75	18.25	Purplish-pink	18.30
9	Carvacrophthalein	16.65	18.05	"	18.15
10	Octobromophenolphthalein	16.45	17.95	Purple	17.85
11	Thymoltetrabromophthalein	16.05	17.40	Bluish-purple	17.25
12	α -naphtholphthalein	15.10	15.80	Bluish-green	15.82
13	α -naphtholtetrabromophthalein	14.75	15.30	Green	15.30
B. Fluoresceins: constant 4.7.					
1	3-oxyfluorescein	20.45	23.8	Yellow	23.6
2	Phloroglucinphthalein	20.15	23.1	"	23.2
3	Fluorescein	20.05	23.2	"	23.0
4	Resorcinbenzeine	19.95	22.6	"	22.9
5	Iso eosine	19.90	22.8	Yellowish-salmon	22.8
6	Eosine	19.35	21.9	"	22.0
7	Erythrosine	19.05	21.35	"	21.5
8	Galleine	18.4	21.5	"	20.6
C. Some other Fuchsone-derivatives which do not follow the above law.					
A.	Aurine	18.90	20.8	Salmon	—
B.	Benzaurine	17.55	21.7	Yellow-salmon	—
C.	Diethylrhodamine	18.10	21.8	"	—
D.	Quinolnphthalein	20.25	20.1	Salmon	—
E.	Resorcinolphenylphthalein	Violet only	20.25	"	—

The absorption bands in H_2SO_4 of the following substances are: Fluorane, 20.05; fuchsone, 22.0; benzophenone, 21.7; benz-naphthaurine and its $-CO_2H$ derivative (from BBA and naphthol), both 17.6.

and, what is remarkable, that the intensity of the colour is about five times as great as when they are dissolved in alkali. The colours obtained in the two cases are *not* identical; thus, all the phthaleins giving a blue colour in alkali give a pink colour in H_2SO_4 . In addition, many other substances of

the fuchsone family can be investigated in H_2SO_4 solution which are not soluble in alkali—*e.g.* fluorane. Table III gives some of the observed results.

The author has been able to trace the very simple law which governs the depression of the colour by H_2SO_4 as compared with alkali. The expression $y = \frac{3}{2}(x - 4.55)$, in which x is the frequency corresponding to alkali solution and y is the frequency corresponding to H_2SO_4 solution for the same phthalein, is found to fit all the direct derivatives of phenolphthalein, and, with the trifling modification to $y = \frac{3}{2}(x - 4.7)$, also fits the whole of the fluorescein series. This means that when the values for y and x actually observed are plotted against one another in the case of all the 50 phthaleins and fluoresceins, all the 50 dots lie approximately on one line. The fact that the quotient is $\frac{3}{2}$ seems to indicate that, as the vibration is $1\frac{1}{2}$ times as fast in H_2SO_4 , there must be three phenyl rings vibrating in H_2SO_4 solution as against two only in alkaline solution. Extraordinary as this may seem, there appears to be no other possible explanation of the raising of the frequency by loading with H_2SO_4 .

It is, however, just possible that the bands in H_2SO_4 are not the bands of alkaline solution raised in frequency, but are secondary bands (originally in the ultra-violet) lowered in frequency by loading with H_2SO_4 . Such secondary bands in alkaline solution, if they exist, would probably have either one-half or one-third of the wave-length of the visible bands, so that the above-discovered law of a linear relationship between H_2SO_4 and alkali solutions would still hold true.

The constant 4.55 (or 4.7) probably expresses the effect of the loading with H_2SO_4 , regarded merely as increasing each of the molecular weights by 98.

NOTE ON AN EXPANSION OF THE PRODUCT OF TWO OBLONG ARRAYS.

BY SIR THOMAS MUIR, LL.D.

(1) The long-known expansion of the product of two oblong arrays takes the form of a sum of products of pairs of determinants. The expansion here brought forward takes on the other hand the form of an aggregate of *single* determinants. Of these last the first is itself a product-determinant, and through being bordered gives rise to all the others. Taking the case of two 3-by-5 arrays we have as an example of it

$$\begin{vmatrix} a_1 & a_2 & \dots & a_5 \\ b_1 & b_2 & \dots & b_5 \\ c_1 & c_2 & \dots & c_5 \end{vmatrix} \cdot \begin{vmatrix} a_1 & a_2 & \dots & a_5 \\ \beta_1 & \beta_2 & \dots & \beta_5 \\ \gamma_1 & \gamma_2 & \dots & \gamma_5 \end{vmatrix} \\ = \begin{vmatrix} P_1 & P_2 & P_3 \\ Q_1 & Q_2 & Q_3 \\ R_1 & R_2 & R_3 \end{vmatrix} - \begin{vmatrix} P_1 & P_2 & P_3 & a_4 \\ Q_1 & Q_2 & Q_3 & b_4 \\ R_1 & R_2 & R_3 & c_4 \\ a_4 & \beta_4 & \gamma_4 & . \end{vmatrix} - \begin{vmatrix} P_1 & P_2 & P_3 & a_5 \\ Q_1 & Q_2 & Q_3 & b_5 \\ R_1 & R_2 & R_3 & c_5 \\ a_5 & \gamma_5 & \beta_5 & . \end{vmatrix} + \begin{vmatrix} P_1 & P_2 & P_3 & a_4 & a_5 \\ Q_1 & Q_2 & Q_3 & b_4 & b_5 \\ R_1 & R_2 & R_3 & c_4 & c_5 \\ a_4 & \beta_4 & \gamma_4 & . & . \\ a_5 & \beta_5 & \gamma_5 & . & . \end{vmatrix}$$

where $|P_1 Q_2 R_3|$ is the product of $|a_1 b_2 c_3|$ and $|a_1 \beta_2 \gamma_3|$. Had the number of given columns being *six* instead of five, there would have been four additional terms in the expansion, all of them involving the elements with the suffix 6, one like those already obtained of the 4th order, two like those of the 5th order, and one of the 6th order namely,

$$\begin{vmatrix} P_1 & P_2 & P_3 & a_4 & a_5 & a_6 \\ Q_1 & Q_2 & Q_3 & b_4 & b_5 & b_6 \\ R_1 & R_2 & R_3 & c_4 & c_5 & c_6 \\ a_4 & a_5 & a_6 & . & . & . \\ \beta_4 & \beta_5 & \beta_6 & . & . & . \\ \gamma_4 & \gamma_5 & \gamma_6 & . & . & . \end{vmatrix} .$$

The proof is quite simple. We have only to note in the first place that the product of the two arrays is expressible as a single determinant

$$\begin{vmatrix} P_1 & P_2 & P_3 & a_4 & a_5 \\ Q_1 & Q_2 & Q_3 & b_4 & b_5 \\ R_1 & R_2 & R_3 & c_4 & c_5 \\ a_4 & \beta_4 & \gamma_4 & -1 & . \\ a_5 & \beta_5 & \gamma_5 & . & -1 \end{vmatrix},$$

the latter being transformable into the former by adding to each of the first three rows a_4 times the 4th row and a_5 times the 5th row. Then we partition this determinant into four, namely, one representing all the terms of it containing both of the elements in its (4,4)th and (5,5)th places, one representing all the terms containing one of these elements without the other, and one representing all the terms containing neither.

(2) The general theorem may be formulated thus:

The product of two m-by-n arrays A, B, is expressible as an aggregate of single determinants, the first of which is the product of the first k columns of the arrays, and the others are formed from this by bordering, namely, bordering first in every way with one of the remaining columns from A and the corresponding column from B, secondly, with two of the remaining columns from A and the corresponding two from B, and so on, those having an odd number of lines in the border being negative and the others positive.

The number of terms in the expansion is evidently

$$(n-k)0 + (n-k)_1 + \dots + (n-k)_{n-k} \\ \text{i. e. } 2^{n-k}$$

(3) The relation between the old expansion and the new is not at all complicated, each term in the latter being the equivalent of a group of terms in the former. Thus, in the example with which we started, where $n, m, k = 5, 3, 3$, the apportionment of the 10 product-terms of the old expansion among the four terms of the new expansion is

$$1 + 4 + 4 + 1;$$

and when $n, m, k = 6, 3, 3$ the apportionment of the 20 among the 8 (that is, 6_3 among 2^{6-3}) is

$$1 + 3 + 3 + 3 + 3 + 3 + 3 + 1.$$

If, further, we group the latter terms according to the number of lines in a border, this takes the form

$$1 + 3 \cdot 3 + 3 \cdot 3 + 1,$$

and we have the verification provided by the known theorem regarding

the expression of a combinatorial as a sum of products of pairs of combinatorials, namely,

$$(n)_m = (k)_m + (n-k)_1(k)_{m-1} + (n-k)_2(k)_{m-2} + \dots$$

(4) The single determinant used at the end of §1 as the equivalent of a product of two oblong arrays is historically interesting. Its counterpart, the similar expression for two *square* arrays, was first used by Spottiswoode in 1853 in bringing forward* Sylvester's theorem of the year before in regard to the multiplicity of form of the product of two determinants; and as we have pointed out elsewhere† an extreme case of it must have been used by Sylvester himself, namely, the case

$$\begin{vmatrix} a_1 & a_2 & \dots & a_5 \\ b_1 & b_2 & \dots & b_5 \\ c_1 & c_2 & \dots & c_5 \end{vmatrix} \cdot \begin{vmatrix} a_1 & a_2 & \dots & a_5 \\ \beta_1 & \beta_2 & \dots & \beta_5 \\ \gamma_1 & \gamma_2 & \dots & \gamma_5 \end{vmatrix} = \begin{vmatrix} . & . & . & a_1 & a_2 & \dots & a_5 \\ . & . & . & b_1 & b_2 & \dots & b_5 \\ . & . & . & c_1 & c_2 & \dots & c_5 \\ a_1 & \beta_1 & \gamma_1 & -1 & . & \dots & . \\ a_2 & \beta_2 & \gamma_2 & . & -1 & \dots & . \\ . & . & . & . & . & . & . \\ a_5 & \beta_5 & \gamma_5 & . & . & \dots & -1 \end{vmatrix}.$$

With this before us it is important to note that this expression for the product of two oblong arrays and the Binet-Cauchy expression of 1812 are the two extremes of a series of such expressions, and that the one used by us at the end of §1 is an example of the *third* of the series.‡

CAPE TOWN, S.A.; March 28, 1917.

* *Crelle's Journ.*, li, pp., 238-248.

† *Hist. of Dets.*, ii, pp., 199-200.

‡ Other related papers are:

Muir, T., "On a determinant formed by bordering the product of two determinants," *Messenger of Math.*, xi (1882), pp. 161-165.

Nanson, E. J., "On partial compounds," *Messenger of Math.*, xxvii (1898), pp. 17-19.

A SUMMARY OF THE DISTRIBUTION OF THE GENERA OF SOUTH AFRICAN FLOWERING PLANTS.

(WITH SPECIAL REFERENCE TO THOSE FOUND IN THE DIVISIONS OF
UITENHAGE AND PORT ELIZABETH.)

BY S. SCHÖNLAND.

INTRODUCTORY REMARKS.

I have often felt the need of a concise account of the distribution of the genera of Phanerogams occurring in South Africa. I felt this need especially when writing an account of the Flora of Uitenhage and Port Elizabeth, which is nearing completion. As such a summary, based on recent data, was not available, there was nothing left but to prepare it myself. It was suggested to me that it might also be of use to other botanists, and I, therefore, offer it for publication.

It has been my endeavour to show as much as possible the general trend of distribution rather than give the exact area of each genus. Moreover, large parts of South Africa are so incompletely known botanically that a more detailed account would in many cases have given a wrong impression. However, where details are given, they may be taken to be as accurate as available records, checked by my own personal knowledge, have enabled me to make them. The treatment of the genera in different families is somewhat uneven. This is due to various causes. In some large orders references to more detailed accounts are readily available, while in others records are very meagre; but, above all, it has been my aim to focus the whole account on the Flora of Port Elizabeth and Uitenhage, which in many cases actually or approximately forms the boundary of genera, while in others it marks the area where the full force of the genera spreading eastwards from the South-West and of others spreading from the East to the West along the southern coast belt is broken. It will be noticed that few genera of fairly general distribution in South Africa are absent from it. Orders and genera not coming near this area were only dealt with for completeness' sake. Some interesting points which have disclosed themselves while preparing this paper will be dealt with in the account of the Flora of Uitenhage and Port Elizabeth, which I hope to be able to submit to the

Society shortly. The arrangement of the orders is in accordance with Engler's system.

With few exceptions, I have abstained from any reference to the occurrence, if any, of our genera outside temperate South Africa, as this can easily be gleaned from the work of Thonner, 'Die Blütenpflanzen Afrikas,' now available in an English translation, and other works of a similar nature, besides works of greater magnitude. I would have preferred to have dealt separately with the genera belonging to different divisions of orders, and it would further have been valuable to have dealt with subgenera and series of species in each genus. Perhaps somebody with more leisure than I can command will take the hint, and give us, at no distant date, a more elaborate account, in which altitudes, ecological zones, geological formations, and similar points are also carefully taken into consideration.

GYMNOSPERMAE.

CYCADALES.

Cycadaceae.

The Uitenhage district is not far from the western limit of the *Cycadaceae* in South Africa. *Stangeria* Th. Moore is endemic from Natal to Tharfield, east of Port Alfred. The only other South African genus, *Encephalartos* Lehm., extends a little west of the Uit. div. A line half-way between the 23rd and 24th degrees of longitude to the neighbourhood of Willowmore is, according to Dr. Rattray, approximately the boundary.

CONIFERAE.

Pinaceae.

The only South African genus, *Widdringtonia* Endl., occurs from Clanwilliam along the coast districts to mountains of the eastern parts. Various species of *Pinus* L., have become naturalised in parts of South Africa, e. g. *Pinus Pinaster* on the hills south of Grahamstown.

Taxaceae.

Podocarpus L'Hér., found from Clanwilliam and the Cape Peninsula through all the forests of the southern coast belt to the East and North-east.

GNETALES.

Gnetaceae.

Represented in South Africa by *Welwitschia* Hook. f. Only found in the western Arid part of Great Namaqualand and northwards to Angola.

ANGIOSPERMAE.

MONOCOTYLEDONEAE.

PANDANALES.

Typhaceae.

The genus *Typha* L. has not been recorded in the southern coast belt from localities west of Van Stadens. It is found, however, in the Karroo and in the eastern and north-eastern parts.

HELOBIAE.

Hydrocharitaceae.

This order is represented in South Africa only by one species of *Lagarosiphon* Harv. It has its western limits at the Gamtoos River, and extends to Natal and the Transvaal.

Potamogetonaceae.

It is curious that out of five species recorded of *Potamogeton* L. in the Uit. and P. E. div. four have not been collected recently. This may be due to insufficient attention having been paid to them, possibly also to drainage of places where previously conditions for their growth were more favourable. *Zostera* L. occurs in the tidal portion of the Zwartkops River (also recorded from the Kleinriver Vley and from Natal). *Ruppia* L. is only recorded from the Cape div. and *Zannichellia* L. from western and northern parts.

Naiadaceae.

Najas L. is recorded from the Transvaal only.

Aponogetaceae.

Aponogeton Thunb. extends from north-eastern and eastern parts, mainly through the coast districts to the Cape div., but also into the central parts to the Somerset East and Richmond divs.

Scheuchzeriaceae.

Triglochin L. is found in the coast belt from the Cape Peninsula to Natal, going inland to the Paarl and Griqualand East.

GLUMIFLORAE.

Gramineae.

Without taking into account obviously introduced genera almost all genera of more or less general distribution are recorded from the Uit. and P. E. divs. Exceptions are *Anthoxanthum* L. (the species of which have

either a decided eastern or decided western distribution), *Imperata* Cyr., and *Enneavogon* Desv. (recorded in the south-west coast region only from the Knysna). Of the genera of decidedly south-western distribution the following are absent: *Prionanthium* Desv., *Pentameris* Beauv., *Chaetobromus* Nees, *Poa* Stapf, *Trisetum* Pers., *Urochlaena* Nees, while *Brizopyrum* Nees, and *Scleropoa* Griseb. have near Port Elizabeth their eastern limit, and *Aira* L., *Avenastrum* Juss., *Ehrharta* Thunb., *Lasiachloa* Kth., *Vulpia* Gmel. extend still further east. (*Aira* and *Ehrharta* are found also on the mountains of Tropical Africa.) *Atropis angusta* Stapf, which the author thinks has come from "saline places by the Zwartkops River" has not been found by recent collectors. Of the genera which are entirely or mainly Central the following are absent: *Brachypodium* Beauv., *Arundinaria* Mich., *Tetrachne* Nees, *Oropetium* Trin., *Secale* L. There are about twenty-three genera in South Africa which have an exclusively eastern distribution (some extending into the Kalahari region). These are absent with the exception of *Elionurus* Humb. et Bonpl. (reaching its western limit at the Zwartkops R.), *Oplismenus* L. (western limit Springfields, near Uitenhage), *Chloris* Sw. (distribution west of Uitenhage insignificant).

Cyperaceae.

On the whole the delimitation of genera by C. B. Clarke in the 'Flora Capensis' has been followed, though here and there it is very unsatisfactory. Thus *Pycnus* Beauv. is not sharply divided from *Cyperus* Linn., *Fimbristylis* Vahl, and *Bulbostylis* Kunth grade into one another, and *Scirpus* Linn. and *Ficinia* Schrad. should be united.

- (a) Fairly generally distributed (all represented in the Uit. and P. E. divs.):

Kyllinga Rottb., *Pycnus* Beauv., *Cyperus* L., *Fimbristylis* Vahl, *Scirpus* L., *Carex* L. The following are absent from the western region and western parts of central region: *Mariscus* Gaertn., *Eleocharis* R. Br., *Bulbostylis* Kunth, *Ficinia* Schrad., *Fuirena* Rottb., *Schoenoxiphium* Nees.

- (b) Mainly northern and north-eastern:

Lipocarpa R. Br., *Rhynchospora* Vahl (also found at Grahamstown and on Table Mountain), *Costularia* C.B.Cl., *Cladium* R. Br. (also at Van Stadens and Krakakamma), *Scleria* Berg. (extends to Komgha division), *Eriophorum* L., and *Eriopora* A. Rich. (Transvaal only).

- (c) Mainly or entirely in the south-west

Carpia R. Br. (extends to Somerset East and Pondoland), *Ecklonea* Steud. (Cape Peninsula and Bains Kloof), *Epischoenus* C.B.Cl. (Cape Peninsula to Riversdale), *Tetraria* Beauv. (extends to Somerset East

and Pondoland), *Macrochaetium* Steud. (Swellendam to Van Stadens), *Chrysithrix* L. (Cape Peninsula to Riversdale).

There remains *Schoenus* L. recorded from the southern coast districts to Komgha and also from Griqualand West, but not from the Uitenhage and Port Elizabeth divisions.

PRINCIPES.

Palmae.

There are only three genera of palms found in the south-eastern coast strip. *Hyphaene* Gaertn. does not go south of Natal, *Jubaeopsis* Becc. is found in Pondoland. *Phoenix* L. extends to the east bank of the mouth of the Bushmansriver, thus coming near the P. E. div.

SPATHIFLORAE.

Aroideae.

The two tropical genera, *Stylochiton* Lepr. and *Ptisar* L. extend to Natal. *Zantedeschia* Spreng. (*Richardia* Kunth) has chiefly north-eastern and eastern distribution, extending to Aliwal North and in the coast belt to the Cape Peninsula.

Lemnaceae.

It is somewhat strange that this order is not recorded west of Port Elizabeth. Both *Lemna* L. and *Wolffia* Hook. are found in the coast belt from there to the Tropics.

FARINOSAE.

Flagellariaceae.

Flagellaria L. extends from the Tropics in the coast districts to East London.

Restiaceae.

There are many more *Restiaceae* found in the Uitenhage and Port Elizabeth divisions than are recorded in the 'Flora Capensis.' Most of them are found either in the hills bordering the districts on the west or close to the sea. In the Albany and Bathurst districts only eight species have been found, whereas here we have about twenty-five. In Natal *Restio* L. and *Leptocarpus* R. Br. are represented by one species. It is a well-known fact that the large majority of the South African *Restiaceae* are confined to the South-west. The following genera are absent from the Uitenhage and Port Elizabeth divisions: *Askidiospermum* Steud., *Willdenovia* Thunb., *Ceratocarzum* Nees, *Anthochortus* Nees, *Phyllocomos* Mast.

Xyridaceae.

Xyris L. has mainly north-eastern and eastern distribution, extending to Somerset East and Kingwilliamstown. One species appears to have a

disjointed distribution, being also found on the Cape Peninsula and in the Tulbagh division.

Eriocaulaceae.

Eriocaulon L. is found in eastern and north-eastern parts, extending to Kingwilliamstown. *Mesanthemum* Koern. only just crosses the Tropics, while *Paspalanthus* Mast. is recorded from various places in the Transvaal.

Commelinaceae.

Aneilema R. Br. is found in eastern and north-eastern parts to Pondoland. *Coleotrype* R. Br. and *Floscopa* Lour. are recorded from Natal. *Commelina* L. and *Cyanotis* D. Don are absent from the western region and the western part of the central region. The latter does not reach the Cape Peninsula.

Pontederiaceae.

Only one species of *Heteranthera* Ruiz et Pavon is recorded from the Transvaal Bushveldt.

LILIIFLORAE.

Liliaceae.

A. Genera extending more or less into Tropical Africa:

- (a) More or less generally distributed in South Africa (all found in the Uit. and P. E. divs.):

Asparagus L., *Bulbine* L., *Eriospermum* Jacq., *Chlorophytum* Ker., *Anthericum* L., *Tulbaghia* Linn., *Drimia* Jacq., *Dipcadi* Medic., *Albuca* L., *Urginea* Steinh., *Ornithogalum* Linn., *Androcymbium* Willd.

- (b) Mainly or only in the east and north-east:

Smilax L. (to Natal), *Kniphofia* Moench. (to Aliwal North and Uitenhage, one species on the Cape Peninsula), *Dracaena* L. (in the coast districts to Uitenhage), *Sansevieria* Thunb. (to Graaff Reinet and Uit. div.), *Notosceptrum* Kunth. (Transvaal and Natal), *Schizobasis* Bak. (extends to Hopetown and Somerset East, not to Natal), *Eucomis* L'Her. (to Graaff Reinet and Uit. div.), *Scilla* L. to Uit. and Somerset divs.), *Drimiopsis* L. (to East London), *Iphigenia* Kunth. (parts bordering on Tropics to Witbank, Transvaal, not in Natal), *Gloriosa* L. (on the coast as far as Port Alfred), *Littonia* Hook. f. (to Orange Free State and the Bashee), *Walleria* Kirk (Transvaal only).

- (c) Mainly or only in S.W. coast region:

Hyacinthus L. (only Cape Peninsula, Cape Flats, and at Tulbagh), *Caesia* R. Br. (eastwards to Uit. div.), *Ornithoglossum* Salisb. (extends through central region to Uit. div., Cradock, and Bechuanaland), *Wurmbea* Thunb. (extends to Port Elizabeth and in the mountains from Somerset East to Natal).

(d) Mainly in central regions :

Haworthia Duval (radiates in all directions, but not reaching the Cape Peninsula nor Natal).

B. Genera endemic in South Africa (Daubinya Lindl., locality unrecorded) :

(a) Eastern and north-eastern :

Behnia Didr. and *Agapanthus* L'Her. (westwards to Somerset East and Uit. div.), *Galtonia* Dene. (to the Transkei and Aliwal North), *Litanthus* Hook. f. (from the Transvaal and Swaziland to Somerset East and Uit. div., but not recorded from Natal), *Bowiea* Harv. (to Katberg), *Sandersonia* Hook. f. (Pondoland, Griqualand East, Natal).

(b) Southern coast districts :

Veltheimia Gled. (Little Namaqualand to Kaffraria), *Nanolirion* Benth. (Tulbagh div. only), *Chamaealoe* Berg. (Uit. div.), *Neodregea* C. H. Wright (P. E. and Grahamstown), *Neopatersonia* Schönl. (Uit. div.), *Lachenalia* Macq. (mainly S.W., extending to western and central regions, one in the Transkei), *Dipidaz* Salisb. (S.W. to Port Elizabeth), *Baeometra* Salisb. (S.W. to Riversdale).

(c) Mainly in central districts and extending in various directions (all in Uit. div.):

Massonia Thunb., *Polyzema* Kunth., *Gasteria* Duv., *Apicra* Willd. (none reaching the Cape Peninsula nor Natal with the exception of *Gasteria*, which is found in Natal). *Apicra* Willd. is not recorded from the Uit. and P. E. divs.

(d) Mainly western :

Whiteheadia Harv. (Namaqualand and south of Graaff Reinet), *Pseudogaltonia* O.K. (Great Namaqualand).

There remain only *Bulbinella* Kunth. with fairly general distribution in South Africa, though not found on the Cape Peninsula (absent from Tropical Africa, but found in New Zealand and the Campbell and Auckland Islands), *Rhadamanthus* Salisb. with rather erratic distribution (one in western Karroo, one near Grahamstown and East London, one doubtfully belonging to the genus at Komgha).

Juncaceae.

Juncus L. is found in all South African regions, but the number of species is particularly large (twelve) in the Uitenhage and Port Elizabeth divisions. The south-western genus *Pronium* E. Mey. is found still a little further east; the third genus *Luzula* De. has only been found on the Katberg, at Cathcart, and on the Mont aux Sources.

Haemodoraceae.

Barberetta Harv. is only known from Natal and the Transkei. The south-western genus *Dilatria* Berg. extends to the Riversdale division. *Wachendorfia* L. has its eastern limit in the Uitenhage district, while *Lanaria* Ait. extends to the east of Port Alfred, as does also *Cyanella* L. As regards *Pauridia* (placed by some under *Haemodoraceae*), see under *Amaryllidaceae*.

Amaryllidaceae.

The number of genera of *Amaryllidaceae* endemic in South Africa exceeds the number of those also found beyond its boundaries. All genera of fairly general distribution are found in the Uit. and P. E. divs.

(a) Fairly generally distributed:

Brunsvigia Heist., *Crinum* L. (the species are mainly eastern). *Hypoxis* L. (absent in central parts except towards the East), *Forbesia* Eckl. (*Curculigo* Gaertn.), *Nerine* Herb. and *Ammocharis* Herb. (both extend to Damaraland).

(b) Mainly western and south-western, all endemic:

Pauridia Harv. (to Humansdorp), *Ianthe* Salisb. (to Grahamstown), *Hessea* Herb. (to Uitenhage, Graaff Reinet and Hopetown), *Carpolyza* Salisb. (Cape Peninsula only), *Strumaria* Jacq. (extends to Uitenhage and into central parts, not on Cape Peninsula), *Amaryllis* L. (S.W. only), *Gethyllis* L. (to Uitenhage and Graaff Reinet), *Vallota* Bak. (neighbourhood of George only).

(c) Mainly eastern and north-eastern (see also *Crinum* L. above):

Buphane Herb. and *Haemanthus* L. (both to Graaff Reinet and in the coast belt to Malmesbury), *Cyrtanthus* Ait. (in the coast belt to Caledon) and the following endemic genera: *Clivia* L. (to south of Grahamstown), *Apodolirion* Bak. (to Graaff Reinet and Uitenhage), *Anoiganthus* Bak. (Swaziland to East London and Somerset East), *Rhodohypoxis* Nel. (Griqualand East and Natal).

Velloziaceae.

Barbacenia Vaud. (*Vellozia* Vaud.) is only found in north-eastern and eastern parts to Griqualand East and Pondoland.

Dioscoreaceae.

Dioscorea L. and *Testudinaria* Salisb. have mainly north-eastern and eastern distribution. The first extends in the coast belt to George; the second, which is endemic, to Somerset East and Humansdorp. The distribution of *Dioscorea* in the southern parts is very erratic. There is one species growing at Kamaehs, near Uitenhage.

Iridaceae.

There are a remarkable number of genera endemic in South Africa. Of these only *Homeria* Vent. is fairly generally distributed. *Babiana* Ker is found in South Africa and the island of Socotra, *Geissorhiza* Ker in South Africa and Madagascar, *Watsonia* Mill. in South Africa, Madagascar, and the Mascarene Islands. (See also *Ferraria* and *Romulea*.)

(a) Fairly generally distributed:

Moraea L., *Homeria* Vent. (absent from the greater part of the central region and the western region), *Aristea* Ait., *Hesperantha* Ker., *Lapeyrousia* Pouv. (not recorded from the Uit. and P. E. divs., but in Bathurst div.), *Watsonia* Mill. and *Acidanthera* Hochst. (absent from the central and western parts), *Tritonia* Ker, *Gladiolus* L. (absent from the greater part of the central region), *Antholyza* L. (absent from central and western region).

(b) Mainly or entirely south-western:

Ferraria L. (to Uit. div., one species at Victoria West, the only genus of this group besides *Romulea* Maratti extending to Tropical Africa), *Galaxia* Thunb. (not reaching Uit. div.), *Bobartia* Ker. (to East London), *Witsenia* Vent. (to Swellendam), *Cleanthe* Salisb. (only recorded from the Paarl and Paardeberg), *Nivenia* Vent. (Caledon to Oudtshoorn), *Klattia* Bak. (Swellendam and Riversdale), *Geissorhiza* Ker and *Ixia* L. (to Griqualand East and Natal?), *Pillansia* L. Bol. (only in Palmiet River mountains), *Streptanthera* Sw. (only recorded from Tulbagh), *Micranthus* Pers. (to Uit. div.), *Freesia* Klatt (Swellendam to Grahamstown), *Melasphaerula* Ker (to Uit. div.), *Sparaxis* Ker (to Uit. div.), *Synnotia* Sw. (Clanwilliam to Cape Peninsula), *Romulea* Mar. (to Grahamstown, but again in various parts of Tropical Africa and in the Mediterranean region).

(c) Mainly central and western:

Syringodea Hook. f. (endemic, recorded from Uit. div.).

(d) Mainly or entirely eastern:

Schizostylis Backh. et Harv. (endemic, to Stockenström division), *Dierama* K. Koch (to Somerset East and Humansdorp div.), *Crocasmia* Planch. (to Tembuland).—(*Keitia* Reg., supposed to come from Natal, is probably not of South African origin.)

SCITAMINEAE.**Musaceae.**

A native species of *Musa* L. is found in the Northern Transvaal. *Strelitzia* L. is endemic and occurs from the Knysna district to Natal. At both extreme points of distribution only *S. augusta* occurs, whereas *S. Reginae*

is found from Humansdorp to Pondoland. One species appears to be confined to the Uit. div.

Cannaceae.

Canna indica has become naturalised in the Van Stadens Pass.

Zingiberaceae.

Kaempferia L. (including *Siphonochilus* Wood et Franks) extends into Zululand and Natal.

MICROSPERMAE.

Burmanniaceae.

Burmannia, L. only just reaches temperate South Africa in the north-east at Inhambane.

Orchidaceae.

Few genera are represented in the Western Region and few enter the Central Region in its eastern and western parts. There are a number of endemic genera of which only one is restricted to the north-eastern and eastern parts, namely *Huttonaea* Harv. (Transvaal, Natal to Transkei). Most of the endemic genera are entirely or mainly found in the south-west: *Acrolophia* Pfitz. (extends to Somerset East div. and Komgha), *Bartholina* R. Br. (from Cape Peninsula to Grahamstown), *Satyridium* Lindl. (Cape, Caledon, and Worcester divs.), *Avicaps*, Lindl. (South-west and Little Namaqualand, not on Cape Peninsula), *Pachites* Lindl. (Cape Peninsula to Riversdale), *Orthopenthea* Rolfe (Clanwilliam div. to Humansdorp div., Ceres div.), *Monadenia* Lindl. (Little Namaqualand, South-west to Natal and the Transvaal), *Amphigena* (Cape and Caledon divs.), *Forficaria* Lindl. (Worcester, Stellenbosch, and Knysna divs.), *Penthea* Lindl. (Clanwilliam to George, one species also on the Zuurberg in the Alexandria div., and in Natal), *Schizodium* Lindl. (South-west to Port Elizabeth, also in Van Rhynsdorp div.), *Ceratandra* Rolfe (Clanwilliam div. to the Knysna), *Ceratandropsis* Rolfe (Clanwilliam div. to Grahamstown), *Evota* Rolfe (Piquetberg, Tulbagh, Cape, Worcester, and Ceres divs.), *Ommatodium* Lindl. (Little Namaqualand, Clanwilliam to Swellendam), *Anochilus* Rolfe (Little Namaqualand, South-west, but not on Cape Peninsula, one species in the Molteno div.), *Corycium* Sw. (Coast belt from Little Namaqualand through Clanwilliam to the East and North-east, one species in the Ceres div., two in the Cradock div.). Amongst the genera which are also found in Tropical Africa, there are:

(1) Some with very restricted distribution: *Corymbis* Thouars (Pondoland), *Zeuxine* Lindl., *Platylophus* A. Rich., *Peristylus* Blume and *Cynorchis* Thouars (Natal), *Pogonia* Juss. (Transvaal).

(2) A number with more or less extended distribution in north-eastern and eastern parts (many going westwards in the coast belt) :

(a) Not beyond the Kei River :

Megaclinum, Lindl. (to Pondoland), *Ansellia* Lindl. (to Natal).

(b) Not beyond the Uit. div. :

Calanthe R. Br. (to Pirie Forest), *Lissochilus* R. Br. (to Somerset East and Uit. divs.), *Brachycorythis* Lindl. (to Bedford div., absent from the Albany and Bathurst divs.), *Schizochilus* Sond. (to Fort Beaufort div., absent from the Albany and Bathurst divs.), *Stenoglottis* Lindl. (to Grahamstown).

(c) Not beyond the Swellendam div. :

Polystachya Hook. (to the Knysna), *Angraecum* Bory (to Somerset East and Swellendam divs.), *Listrostachys* Reichb. f. (to Somerset East and the Knysna), *Mystacidium* Lindl. (to the Bedford div. and the Knysna), *Platanthera* L. C. Rich. (one species in Natal and the Transvaal and two from Grahamstown to Riversdale), *Habenaria* Willd. (to Somerset East and Swellendam divs.), *Brownleea* Harv. (to George div.).

(d) Not beyond the Clanwilliam div. :

Liparis L. C. Rich. (to Cape Peninsula), *Eulophia* R. Br. (to Aliwal North, Bedford, and along the coast belt to Clanwilliam div.), *Bonatea* Willd. (to Somerset East div. and Malmesbury div., not on Cape Peninsula), *Satyrion* Sw. (to Cradock, Somerset East, Malmesbury, and Piquetberg divs.), *Herachelia* Lindl. (almost restricted to temperate South Africa, one species in Tembuland and the Transvaal, Somerset East div., and from Grahamstown to George, but mainly south-western to Clanwilliam, one on Zwarteberg Pass near Prince Albert), *Disa* Berg. (mainly South African to Somerset East div. and in the coast belt to Clanwilliam div., but also in Calvinia and Ceres divs.), *Pterygodium* Sw. (almost restricted to temperate South Africa, to Transkei and then again from Grahamstown to the Clanwilliam div., also in the Ceres div.), *Disperis* Sw. (to Somerset East and along the coast belt to Clanwilliam div., also in the Ceres div.).

DICOTYLEDONEAE.

ARCHICHLAMYDEAE

(Choripetalae and Apetalae).

PIPERALES.

Piperaceae.

The only well known species of *Piper* L. is found in the Swellendam div.,

the Knysna div., Pondoland, Natal, and the Transvaal. *Peperomia* Ruiz et Pavon has mainly eastern and north-eastern distribution extending to Somerset East and in the coast belt to the Cape Peninsula.

SALICALES.

Salicaceae.

Only *Salix* L. is found in South Africa, represented in the Uit. and P. E. divs. by a widely distributed species. Two other species are found in the Transvaal and Natal respectively.

MYRICALES.

Myricaceae.

Myrica L. is widely spread in the southern coast districts of South Africa.

URTIALES.

Ulmaceae.

The South African genera are chiefly of eastern or north-eastern distribution. *Celtis* L., represented by only one species, extends to Graaff Reinet and in the forests of the southern coast belt to the Cape Peninsula. *Trema* Lour. (*Sponia* Comm.) extends to Kentani and *Chaetacme* Planch. to Grahamstown.

Moraceae.

Ficus L. is the only genus native in South Africa. It radiates from the Tropics to the mountains and the eastern borders of the Central Region, and in the coast belt to the west of the Uitenhage div., but does not reach the Cape Peninsula.

Urticaceae.

Most South African genera radiate from the Tropics through the north-eastern and eastern parts, only one native species of *Urtica* L. reaching the Uit. and P. E. divs. (not the Cape Peninsula).

Australina Gaud. extends to the Cape Peninsula. *Urera* Gaud. extends to Komgha; *Flourya* Gaud. to Somerset East, Grahamstown and Port Alfred; *Pouzolzia* Gaud. to Komati Poort and P. P. rust and Natal.

Forskohlia L. extends from Damaraland through Bushmanland to Graaff Reinet. I have no South African record of *Droguetia* Gaud., which Thonner states is found in South Africa.

PROTEALES.

Proteaceae.

To anybody acquainted with the general distribution of *Proteaceae* in South Africa it will not be surprising that out of fourteen genera in the 'Flora Capensis' only four are recorded in the Uit. and P. E. divs. *Paranomus*, Salisb. et Knight (*Nivenia* R.Br.) finds its eastern limit at

Vanstadens, where most of the recorded species of other genera also occur. *Leucadendron adscendens* R. Br. extends to Grahamstown, while *Leucospermum* R. Br. in one species only (*L. attenuatum* R. Br.) extends to Gazaland, and *Protea* L. has a wide eastern distribution, though with only few species. Some of the forests in our districts will probably be found to include a representative of the genus *Faurea* Harv., which occurs at the Knysna and then again further east from the Transkei onwards. Sim records it on the authority of Mr. McNaughton from Zitzikamma. *Hakea* Schrad. is naturalised near Grahamstown.

SANTALALES.

Santalaceae.

All genera of South African *Santalaceae* are represented in the Uit. and P. E. divs. (for the genus *Grubbia* see *Grubbiaceae*). The genus *Thesidium*, which, on the whole, is of south-western distribution, extends eastwards only as far as the neighbourhood of Grahamstown. The distribution of the other four genera is much wider and does not call for any remarks. They are *Thesium* L., *Colpoön* Berg. (*Rhoiocarpus* A. DC.—endemic), *Osyris* L., *Osyridicarpus*, A. DC.

Grubbiaceae (endemic in S.W. Cape Colony).

The only genus, *Grubbia* Berg., is found from Clanwilliam to George. In the 'Flora Capensis,' xi, p. 327, Sonder also records *Grubbia stricta* DC. from mountains in the district of Uitenhage. It has not been found so far east in recent years.

Olacaceae.

Ximenia Plum. is recorded from Natal.

Loranthaceae.

As far as I know, the genus *Loranthus* L. finds its western limit as regards the southern coast districts in the Uit. div., radiating, like so many other South African genera, from Tropical Africa, and penetrating from the north to Namaqualand and Clanwilliam. The genus *Viscum* L. has a more general distribution, our districts yield the most interesting *Viscum minimum* Harv. and *V. crassulae* E. & Z. both extending, however, to the neighbourhood of Grahamstown.

Balanophoraceae.

Of the two South African genera, *Mystroptalon* Harv. is of south-western distribution (from the Caledon district to near Sir Lowry's Pass), while *Sarcophyte sanguinea*, being fairly common in the Albany district, is sure to be found near Uitenhage, though not recorded.

ARISTOLOCHIALES.**Aristolochiaceae.**

One species of *Aristolochia* L. is found in the Northern Transvaal.

Rafflesiaceae.

The occurrence of *Cytinus dioicus* Juss. at Port Elizabeth marks the eastern limit of the genus in the coast districts of South Africa. Westwards it extends to Saldanha Bay. According to Marloth it extends eastwards to Kaffraria, but no locality beyond that mentioned is known to me.

Hydnoraceae.

The wide distribution of *Hydnora africana* Thunb. which is found both west and east (as far as Kingwilliamstown) of Uitenhage, makes it practically certain that it will also be found in our districts, though hitherto unrecorded. *Hydnora* is perhaps unique amongst South African genera, extending so far in the southern coast belt by passing through the western region into the Tropics.

POLYGONALES.**Polygonaceae.**

The genus *Oxygonum* Burch. is absent from the Uit. and P. E. divs., but (leaving out a Stellenbosch record in the 'Flora Capensis' which I consider as doubtful) it is only found in the Kalahari region, Natal and Great Namaqualand. The three other South African genera are generally distributed.

CENTROSPERMAE.**Chenopodiaceae.**

The genera *Roubieva* Moquin, *Kochia* Roth., and *Salsola* L. are not recorded from the Uit. and P. E. divs. The first is only represented in South Africa by an introduced weed; the second is only found in the Karroo region; the third is found also in the Karroo region, but extends to the Kalahari region, and one species was found by Zeyher at Sidbury, near Grahamstown. The other South African genera have fairly general distribution, and are found in the Uit. and P. E. divs.

Amarantaceae.

Very few species of this order are found west of Uitenhage in the coast districts, while they increase in number towards the East and in the northern districts of Cape Colony. *Cyathula* Lour., *Pupalia* Juss., *Achyropsis* Hook. f., *Alternanthera* Forsk., and *Gomphrena* L. are not recorded west of Uitenhage, but as they are weeds they may have been neglected by collectors.

Nyctaginaceae.

Extending from the Tropics into South Africa, only *Phaeoptilum* Radlk.,

having a disjointed distribution, as, in South Africa, it is only known from the Calvinia div. *Pisonia* L. is found in Natal. *Boerhaavia* Vaill. extends to Barkly West, Hopetown and Queenstown. *Mirabilis* L. is introduced in Natal.

Phytolaccaceae.

Adenogramma Reichb., *Limeum* L., *Psammotropha* E. & Z., and *Phytolacca* L. are represented in the Uit. and P. E. divs. *Adenogramma* and *Limeum* have fairly general distribution; the former (endemic) appears to be absent from the central region. *Psammotropha* has mainly western and central distribution, but extends to the Drakensberg in Natal. *Phytolacca* penetrates from the Tropics and reaches Graaff Reinet and the Cape Peninsula. There remain *Microtea* Sw. (Kalahari region to Little Namaqualand), *Giesekia* L. (North and North-east to Graaff Reinet, also in Bushmanland) and *Polpoda* Presl (endemic, restricted to Cape Peninsula).

Aizoaceae.

In the Uit. and P. E. divs. only *Orygia* Forsk., *Hypertelis* E. Mey., *Pharnaceum* L., *Tetragonia* L., *Aizoon* L., *Galenia* L., and *Mesembrianthemum* L. are represented, the last by a large number of species. East of our districts their number is considerably reduced. All these genera are fairly generally distributed in South Africa. *Hypertelis* (endemic) is absent in Natal. *Mollugo* L. has also fairly general distribution, but does not occur on the Cape Peninsula and in the southern coast belt. *Glinus* L. is found in parts adjoining the Tropics and at George. *Sesuvium* L. is found in the West. The remaining genera are endemic: *Anisostigma* Schinz (Great Namaqualand), *Trianthema* L. (Western), *Plinthus* Fenzl (Central), *Coelanthium* E. Mey. (West and South-west), *Acrosanthes* E. & Z. (South-west). (See also *Phytolaccaceae*.)

Portulacaceae.

The genera *Talinum* Adans. (East, North, and North-west) and *Ceraria* Pears. et Stephens (Namaqualand) are absent in the Uit. and P. E. divs. *Anacamperos* Sims, like *Portulacaria* Jacq., a typical Karroo genus, is only represented by species belonging to the section *Telephiastrum* Dill., while *Portulacaria* Jacq. is common in karroid parts of the Uit. div., and mingles even with eastern types.

Basellaceae.

Boussainga *baselloides* has become a weed in hedges, etc., in the Uit. and P. E. divs., as in many other parts of South Africa.

Caryophyllaceae.

The genera *Drymaria* Willd. and *Polycarpaea* Lam. with eastern distribu-

tion do not come near the districts of Uit. and P. E. The other native genera belonging to *Alsinoideae* and *Silenoideae*, which are all widely spread, are recorded. Amongst those with indehiscent fruits only *Scleranthus* L. has not been recorded from the Uit. and P. E. divs., but is sure to be found there. *Pollichia* Soland. occurs in the North and North-east, but not in the central, western, and south-western parts. The Uit. div. appears to be its western boundary. *Herniaria* L. is found in the southern coast belt, the east part of the central region and in northern parts. *Corrigiola* L. occurs in the north-eastern and eastern parts, extending in the southern coast belt to the Cape Peninsula. *Scleranthus* L. is recorded from the southern coast belt and Graaff Reinet.

RANALES.

Nymphaeaceae.

The widely-spread *Nymphaea capensis* Thunb. is not uncommon in the Uit. and P. E. divs.

Ceratophyllaceae.

A species of *Ceratophyllum* L. is recorded from Natal.

Ranunculaceae.

Anemone L. and *Thalictrum* L. have not been found in the Uit. and P. E. divs. The former has a very disjointed distribution in South Africa, the latter is only found east of the Katberg. *Ranunculus aquatilis* L., though not recorded, may be expected to occur in the districts. *R. pubescens* Thunb. is common. *Clematis* L. and *Knowltonia* Salisb., both widely spread in the southern coast belt, extending to the Transvaal, are common in our area.

Menispermaceae.

Antizoma Miers (including *Cissampelos capensis* L.f.) is endemic or just crosses into the Tropics in the West. It is represented in all parts of temperate South Africa, except perhaps in the central part of the central region. The other genera extend from the Tropics: *Cocculus* DC., *Trichlisia* Benth., and *Epinetrum* Hiern (*Synclisia* Benth.) to Delagoa Bay, *Desmonema* Miers and *Jatrochiza* Miers to Natal, *Stephania* Lour. to the Orange Free State and Griqualand East, *Cissampelos* L. to the George div.

Anonaceae.

Only few genera enter from the Tropics along the eastern coast belt. *Popowia* Endl. reaches the neighbourhood of East London, *Anona* L., *Artabotrys* R. Br., and *Uvaria* L. Natal, *Monodora* Dun. Delagoa Bay.

Monimiaceae.

Xymalos Baill. extends from the Tropics into eastern and north-eastern South Africa as far as the Pirie forest near Kingwilliamstown.

Lauraceae.

The genera *Cryptocarya* R. Br. and *Ocotea* Aubl. have a distribution paralleled to a certain extent by that of *Faurea*, there being a more or less wide gap in their distribution in the coast districts; *Ocotea* extends to the Cape Peninsula. Both may be found yet just to reach the Uit. div. in the West. The only species of *Cassytha* recorded extends from the Paarl to Grahamstown. Other species occur much further east.

RHOEADALES.**Papaveraceae.**

Papaver L. (introduced?) and *Argemone* L. (introduced) are each represented in the Uit. and P. E. divs. by one species widely spread in South Africa. *Corydalis* DC. (incl. *Cysticapnos* Adans. [S.W.]) has several native species, of which only one is recorded. *Fumaria* L. (incl. *Discocapnos* Ch. et Schl. [S.W.]) is represented in our area by the introduced *F. officinalis* L. The endemic genus *Trigonocarpus* Schlecht. is only known from the western region.

Capparidaceae.

The herbaceous South African genera of this order belonging to the sub-order *Cleomoideae* are absent in the Uit. and P. E. divs. They are only found in the eastern, north-eastern, and northern parts of South Africa. *Capparis* L., *Cadaba* Forsk, and *Maerua* Forsk are recorded. None of these reach far into the south-west coast region. The genus *Boscia* Lam. probably occurs on the eastern borders of our districts, but is not recorded.

Cruciferae.

Only the genera *Brassica* L., *Sinapis* L., *Heliophila* L., *Nasturtium* R. Br., *Sisymbrium* L., *Lepidium* L., *Capsella* DC. have been recorded from the Uit. and P. E. divs., and since, with the exception of *Heliophila*, these are either entirely or partly represented by introduced species, it will be seen that our area is very deficient in native species of *Cruciferae*, a feature which is characteristic for the whole of South Africa with the exception of the South-West, where the endemic genera, *Carponema* Sond., *Schlechteria* Bol., *Chamira* Thunb., *Palmstruckia* Sond., *Cycloptychis* E. Mey., *Brachycarpaea* DC., are found, also *Heliophila*, which is there enormously developed. In our area there are still thirteen species of *Heliophila*, but the genus thins out further east, four species being recorded from Natal. *Cardamine* L. is fairly common both east and west of our area. The occurrence of *Alyssum* L., *Barbarea* R.Br. in the South-West, evidently native, is

noteworthy. Several genera, represented by introduced weeds such as *Coronopus* Gaertn. (*Senebiera* Poir.), *Rhaphanus* L., *Diplotaxis* DC. are sure to be found yet in our area. *Arabis* L. is recorded from near Graaff Reinet and the mountains of Natal, *Turritis* L. from the Witbergen, and *Matthiola* R. Br. from the eastern parts of the central region to Burghersdorp and the Caledon River.

Resedaceae

are absent in the Uit. and P. E. divs. They are only represented in South Africa by the genus *Oligomeris* Cambess., which extends from the Transvaal to the eastern parts of the Karroo and from Natal to the Fish River Randt.

Moringaceae.

Moringa Juss. occurs in Great Namaqualand.

SARRACENIALES.

Droseraceae.

Drosera L. is found generally distributed in the southern and eastern parts, being absent from the drier interior parts. (*Roridula*, see under *Ochnaceae*).

ROSALES.

Podostomaceae.

Tristicha Thouars. has been found in Calvinia div., Kingwilliamstown div., and in Natal (Pietermaritzburg), and *Sphaerothyli* Boschhoff in Natal.

Hydrostachydaceae.

Hydrostachys Thouars. extends from the Tropics to Natal.

Crassulaceae.

Of the nine genera recognised by Harvey in the 'Flora Capensis,' ii, p. 328, I keep up only six (*Helophytum* E. & Z. and *Bulliarda* DC. being sunk in *Crassula* L. and *Bryophyllum* Salisb. in *Kalanchoe* Adans.). The south-western genus *Grammanthes* DC. is absent in the Uit. and P. E. divs. *Dinaeria* Harv. extends from the Cape Peninsula through our districts to Grahamstown. *Rochea* DC., also an essentially south-western genus, has one species occurring on the Cockscomb Mts., and thus finds its eastern limit in our districts. *Kalanchoe* Adans., which radiates from the Tropics into South Africa without, however (except for a short distance near Norval's Pont) crossing the Orange River in the North, is represented by two species, one of which goes even further west as far as Riversdale. *Cotyledon* L. has a wide distribution all over South Africa. *Crassula* L., which is also universal in South Africa, is largely represented in our area. Several points

of interest come out when the distribution of the species is compared (which is also the case in numerous genera belonging to other orders), but this is beyond the scope of the present paper.

Saxifragaceae.

Of the three South African genera only the widely spread *Montinia* L. f. is recorded from the Uit. and P. E. divs. *Vahlia* Thunb., which has mainly a northern and north-western distribution, is absent, and so is *Choristylis* Harv. with an eastern distribution stopping short at the Katberg.

Pittosporaceae.

Pittosporum Banks extends from the eastern and north-eastern parts in the southern coast belt to George.

Cunoniaceae.

Both *Cunonia* L. and *Platylophus* Don are recorded from the Uit. or P. E. divs. The former is very widely spread in the coast districts of South Africa, while the latter does not reach the Cape Peninsula in the West and does not extend beyond Van Stadens Mountains in the East.

Myrothamnaceae.

Myrothamnus Welw. only occurs in the northern and north-eastern parts to Natal.

Bruniaceae.

This order is almost restricted to the south-west coast region. Three genera only extend to our districts—namely, *Berzelia* Brongn., *Brunia* L., and *Pseudobaekia* T. Anders. A little further east, near Grahamstown, only one species of the first is still to be found. Curiously a species of *Raspalia* Brongn. occurs in East Pondoland and Natal. The order appears to be largely restricted to the Table Mountain sandstone, and this may account for the erratic distribution.

Hamamelidaceae.

Trichocladus Pers., the only South African genus now included in this order, is widely spread east of Uitenhage to the Tropics, and extends westwards to George.

Rosaceae.

Of the ten South African genera only *Alchemilla* L., *Rubus* L., and *Cliffortia* L., are recorded from the Uit. or P. E. divs. The two former have a very wide distribution, while *Cliffortia* is represented in the south-west coast region by a large number of species, thinning out rapidly beyond the Van Stadens River, though extending into Central Africa. Of the remain-

ing genera *Acaena* Vahl., *Grielum* L. have western and north-western distribution, *Potentilla* L. is only found on the Orange River, while all the others are eastern or north-eastern, though *Geum* L. is already found near Grahamstown, *Leucosidea* E. and Z. near Cathcart, and *Pygeum* Gaertn. at Fort Cuninghame.

Leguminosae.

Suborder Mimosaceae.

The genus *Acacia* only is recorded from the Uit. and P. E. divs. It radiates from tropical Africa into temperate South Africa. Two species are still found near Uitenhage, one of which extends also further west, but is only doubtfully native on the Cape Peninsula. All other genera are found in regions adjoining the Tropics, *Elephantorrhiza* Benth. penetrating to the Cradock district.

Suborder Caesalpinaceae.

These have a similar distribution as the *Mimosaceae*. The genera *Cassia* L. and *Schotia* Jacq. are found in the Uit. div., neither of which appear to be recorded west of our districts; *Schotia speciosa* occurs, however, at Mossel Bay.

Suborder Papilionaceae.

These show to a certain extent a similar distribution to the two preceding suborders, and consequently there are a large number of genera only found in regions adjoining the Tropics or penetrating more or less in a south-westerly direction; but, in addition, there is a fairly large number of genera, some represented by an enormous number of species found in all South African floral regions, and, lastly, there are a number of endemic genera, especially in the south-west coast region, some of which are not even nearly related to tropical Africa genera. All genera distributed widely in South Africa are recorded from the Uit. and P. E. divs., some of them—e.g. *Lotononis* Benth., *Aspplathus* L., *Argyrolobium* E. and Z., *Psoralea* L., *Indigofera* L., *Lessertia* L., *Rhynchosia* Lour.—with more or less numerous species.

Amongst the genera which penetrate westward from Natal *Buchenroedera* E. and Z., *Eriosema* E. Mey., *Erythrina* L., and *Calpurnia* E. Mey. find their western limit in our districts. *Lotus* L., and *Dichilus* DC. penetrate to near their eastern borders and may yet be found in them, while *Crotalaria* L. and *Vigna* Lavi penetrate much further west, the former reaching the Cape Peninsula with one species.

Dealing now with the genera of essentially south-western distribution, we find that *Liparia* L., *Euchlora* E. and Z., *Walpersia* Harv., *Hypocalyptus* Thunb., *Loddigesia* Sims, *Viborgia* Thunb., *Fagelia* Neck. are absent.

Cyclopia Vent., *Priestleya* DC., *Borbonia* L., and *Virgilia* Lam. have their eastern limits in our districts. *Podalyria* Lam. extends to Natal; *Amphithalia* Thunb. extends to Cathcart; *Rafnia* E. and Z., and *Lebeckia* Thunb. extend beyond Grahamstown; *Hallia* Thunb. extends to the neighbourhood of Grahamstown.

The genus *Dumasia* DC. is not recorded, although it occurs both east and west—namely, at the Knysna and in the Pirie forest near Kingwilliamstown, in Natal and beyond.

GERANIALES.

Geraniaceae.

The genus *Sarcocaulon* DC. with central, western, and north-western distribution is absent in the Uit. and P. E. divs., although it extends into the Fish River valley. *Erodium* L'Hér is introduced. *Monsonia* L., which is scattered over the greater part of South Africa, is represented by one species. Of *Geranium* L., represented by only few species in the southern coast districts, three species are recorded. The large number of recorded species of *Pelargonium* L'Hér. is only to be expected, as our districts form, in a sense, the centre of the distribution of the genus. The section *Hoarea* DC. does not extend east of the Uit. and P. E. divs. A useful summary of the distribution of the sections of the genus is given by R. Knuth in 'Pflanzenreich,' iv, 129, p. 37.

Oxalidaceae.

Oxalis L., the only genus represented in South Africa, is found in the coast districts of South Africa, with its headquarters in the south-western region. Towards the East the number of species diminish. Few species penetrate towards the interior.

Linaceae.

Linum L. is the only genus represented in South Africa. It is widely spread in the southern coast districts, going far inland in the eastern parts.

Erythroxylaceae.

Erythroxylon P. Browne extends from the Tropics to the eastern and north-eastern parts, and along the east coast to East London.

Zygophyllaceae.

Tribulus Tourn. is very widely spread in South Africa. *Zygophyllum* L. is absent from the eastern parts (my most eastern records are Port Alfred, Fort Brown, Burgersdorp), otherwise it is widely spread with its headquarters in the Karroo. All other South African genera are unrecorded in the Uit. and P. E. divs., namely, *Neoluederitzia* Schinz (Namaqualand), *Seetzenia* R. Br. (western), *Fagonia* Tourn. (western), *Sisyndite* E. Mey.

(north-western), and the Karroid genus *Augea* Thunb.; the latter, however, is almost sure to occur in the north-western parts of the Uitenhage division.

Rutaceae.

All arborescent South African genera are found in the Uit. and P. E. divs. *Calodendron* Thunb. occurs from Swellendam to Natal and beyond. *Clausena* Burm. (incl. *Myaris* Presl) appears to have its western limit near Port Elizabeth, while *Toddalia* Juss. and *Xanthoxylum* L. extend much further west.

Amongst the south-western genera *Empleuridium* Sond., *Adenandra* Bartl. et Wendl., *Phyllosoma* Bol., and *Euchaetis* Bartl. et Wendl. are absent. *Barosma* Willd. and *Agathosma* Willd. are well represented, but thin out further east, only the former reaching Natal. *Coleonema* Bartl. et Wendl. and *Empleurum* Soland. reach in our area their eastern limit. *Diosma* L. is still common near Grahamstown, but does not extend much further east, while *Acmadenia* Bartl. et Wendl. found in the Uit. div. is recorded by Ecklon and Zeyher and Drege from Kaffraria, though it appears to be absent from the intervening strip of country. The occurrence in the Uit. div. of the south-western genus *Macrostylis* Bartl. et Wendl. is doubtful, but in any case it does not extend further east.

Simarubaceae.

The endemic genus *Peglera* Bol. is found in the Kentani district. *Balanites* Del. occurs in extra-tropical Portuguese East Africa.

Burseraceae.

Only *Commiphora* Jacq. (*Balsamodendron* Kunth) extends to South Africa. It occurs in the north-western, northern, north-eastern and eastern parts, in the latter as far south as Kingwilliamstown.

Meliaceae.

Ekebergia Sparm. and *Ptaeroxylon* E. & Z. are found in the Uit. and P. E. divs. The former, according to Sonder, extends to George, but is more frequent east of our districts, while I have no record of *Ptaeroxylon* E. & Z. west of Uitenhage. *Nymania* Lindb. (*Aitonia* Thunb.) is a typical Karroo genus extending to Namaqualand and to the Uit. div. *Trichilia* L. and *Turraea* L. are eastern genera, the latter being found as far west as Port Alfred.

Malpighiaceae.

Four genera extend from the Tropics into the northern and north-eastern parts of South Africa, namely, *Acridocarpus* Guill. et Perr. (to Pondoland), *Sphedamnocarpus* Planch., *Triaspis* Burch. (to Kuruman), *Tristellateia* Thouars.

Polygalaceae.

The only South African genus not found in the Uit. and P. E. divs. is *Securidaca* L., which, however, is only found on the borders of the Tropics in the East. *Mundia* Kunth is a seashore plant extending from the West to the neighbourhood of East London. *Polygala* L. is generally distributed. *Muraltia* Neck. has its headquarters in the S.W. region, extending, however, to Natal and beyond to Nyassaland.

Dichapetalaceae.

Dichapetalum Thouars. (*Chaillatia* DC.) is in South Africa restricted to the Transvaal and Bechuanaland.

Euphorbiaceae (excl. Buxaceae).

There are 42 genera recorded from South Africa, of which only *Euphorbia* L. is universally distributed. Arborescent forms of this genus are still common in the eastern parts of the Uit. and P. E. divs., and seem to have their western limit in the Humansdorp district near Hankey. The order radiates from the Tropics, especially from the North-East and East, gradually thinning out along the southern coast belt, but not penetrating far (apart from *Euphorbia*) into the central parts. Four genera are represented by native species on the Cape Peninsula. In the Uit. and P. E. there are 15, in Natal at least 22.

The following genera are endemic: *Lachnostylis* Turcz. (Swellendam to Port Elizabeth), *Heywoodia* Sim (coast forests of the Transkei and eastern Pondoland), *Toxicodendron* Thunb. (Little Namaqualand), *Seidelia* Baill. (Griqualand West).

The following extend from a north-eastern direction to the West of the Kei River (in addition to *Euphorbia* L.): *Andrachne* W. (Uitenhage, George), *Phyllanthus* L. (to Murraysburg and Uit. div.), *Croton* L. (to Uit. div.), *Jatropha* Thunb. (to Cookhouse and Uit. div.), *Cluytia* L. and *Leidesia* Mull. Arg. (along the coast belt to Cape Peninsula), *Acalypha* L. (to the Uit. div.), *Adenocline* Turcz. (to Hanover, Graaff Reinet, and along the coast belt to the Cape Peninsula), *Gelonium* L. (to P. E. div.), *Ctenomeria* Harv. (to Kei mouth and in the Uit. div.), *Excoecaria* L. (to the Pirie forest).

Mercurialis L. has become naturalised, one species occurring from the Uit. div. to Natal.

Callitrichaceae.

Callitriche L. has one native species in South Africa which is recorded from the Uit. div., the Albany div. and Natal. *C. verna* L. is recorded by Zaklbruckner from the Oliphants River, Clanwilliam.

SAPINDALES (CELASTRALES).

Buxaceae.

Buzus L. is represented by a native species in the coast forests from Alexandria to Pondoland. *Notobuzus* Oliv. occurs from the Transkei to Pondoland.

Anacardiaceae.

The genus *Rhus* L. is generally distributed over South Africa where suitable conditions for its growth prevail. *Loxostylis* Spreng. f. extends from the western parts of the Uit. and P. E. divs. to Natal. *Laurophyllum* Thunb. extends from Van Stadens to Swellendam. *Harpephyllum* Burch. does not grow west of Uitenhage, and extends to Natal. The absence of *Heeria* Meisn. (*Anaphrenium* E. Mey.) is somewhat strange. It extends from the western, north-western and north-eastern parts to the Fish River valley near Grahamstown. The following are also absent: *Protorhus* Engl. (from the Pirie forest eastward), *Smodingium* E. Mey. (Transkei, Basutoland and Natal), *Sclerocarya* Hochst. and *Lannea* Rich. [*Odina* Rosb.] (both eastern and north-eastern).

Aquifoliaceae.

The only South African genus *Ilex* L. occurs from the Cape Peninsula to Natal and beyond.

Celastraceae.

Only *Gymnosporia* W. & A. is generally distributed, the remaining genera are mostly found in the coast belt from the Cape Peninsula to Natal. *Catha* Forsk. (*Methyscophyllum* E. & Z.), though not restricted to South Africa, is in South Africa only found in the Queenstown and Cathcart divs., *Pleurostyliia* W. & A. extends from the Uit. and P. E. divs. to Natal, *Mystroxyton* E. & Z. from the Cape Peninsula to Natal and the Transvaal, *Hartogia* Thunb. (absent from continental tropical Africa, but found in Madagascar) from Riversdale to Uitenhage and Albany, *Elaeodendron* Jacq. from the Gauritz River to Natal and the Transvaal. The following are endemic: *Putterlickia* Endl. (Cape Peninsula, Uit. and P. E. divs. to Natal), *Pterocelastrus* Meisn. (Saldanha Bay to Natal and Transvaal), *Maurocenia* L. (restricted to S.W. Cape Colony), *Lauridia* E. & Z. (Cape Peninsula and from P. E. and Uit. divs. to Kentani, also in Damaraland), *Cassine* L. (Clanwilliam to Natal).

Hippocrateaceae.

Salacia L. is found in Natal and the Transvaal, *Hippocratea* L. on the borders of the Tropics and at Kentani.

Icacinaeae.

I have no record from the Uit. or P. E. divs. of *Cassinopsis* Sond.

C. capensis Sond. occurs according to Sim throughout the forest regions from Grootvadersbosch to Natal. The one species of *Apodytes* E. Mey. which extends throughout the forest regions of Cape Colony and Natal, but does not reach the Cape Peninsula, is found in the Uit. div.

Sapindaceae.

An essentially tropical order which, as defined by Radlkofer and others, is absent from the Cape Peninsula, and is represented by *Dodonaea* L. on the south-western and western mountains, in the Karroo, Namaqualand and Natal. *Allophylus* L. (*Schmidelia* L.) extends in the coast districts from the Knysna to Natal, *Smelophyllum* Radl. from Langekloof to Port Elizabeth. *Pappea* E. & Z. is found in the scrub of Uitenhage, Albany, and Namaqualand, one species is eastern. *Hippobromus* E. & Z. is found in the eastern scrubs and forests and does not seem to occur west of the Uit. div. *Cardiospermum* L. is eastern from the Kei River onwards, *Erythrophysa* E. Mey. is confined to Namaqualand. (For *Ptaerozydon* E. & Z. and *Nymanina* Lindb. [*Aitonia* Thunb.] see under *Meliaceae*.)

Meliantaceae.

There are three genera only: *Greyia* Hook. & Harv. and *Bersama* Fres. (*Natalia* Hochst.) are both eastern, the former confined to South Africa and found from Dohne and Komgha in the mountains to Barberton, the latter from Komgha eastwards. On the other hand, *Melanthus* L. has a decidedly western and central distribution. It is found on the Cape Peninsula, and in the east it does not appear to go beyond East London.

Balsaminaceae.

Impatiens L. is found on river banks in woods from George to Natal and the Transvaal. It has also been collected by Schlechter in the West on the Silver River, a locality unknown to me.

RHAMNALES.

Rhamnaceae.

Zizyphus Juss., though pretty generally distributed in South Africa, has not been recorded in the Uit. and P. E. divs. It is also absent from the Cape Peninsula. *Scutia* Brongn. is found throughout the forest regions of Cape Colony and Natal and reaches the Cape Peninsula. *Rhamnus* L. has a similar distribution, but is also found in the Orange Free State and the Transvaal, but is absent from the Cape Peninsula. *Noltea* Reichb. f. occurs from the Cape Peninsula to Natal. *Helinus* E. Mey. is an eastern genus reaching its western limit on the Zuurberg. *Phyllica* L., is, on the other hand, a typical south-western genus which almost stops at Port Elizabeth, only a few species being found further east as far as Natal. There remains only *Marlothia* Engl., which is only found in Bechuanaland.

Vitaceae (Ampelidaceae).

Rhoicissus Planch. (formerly placed under *Cissus* L. or *Vitis* Tourn.) is found in the southern coast districts of South Africa, one on the Cape Peninsula, chiefly in forests. *Cissus* L. is widely spread in South Africa, but absent from South-West Cape Colony.

MALVALES.**Tiliaceae.**

Grewia L. is fairly generally distributed in South Africa (absent, however, from the greater part of the Karroo), though only poor in species as compared with tropical Africa. *Sparmannia* L. has a well-known species at Stormsriver, George, Knysna, and in the Uit. div., and then occurs again from the Transkei eastwards, *Corchorus* L. and *Triumfetta* L. are essentially tropical genera, but extend into the parts of South Africa adjoining the Tropics, *Triumfetta* L. reaching the Kei mouth.

Malvaceae.

Malva L. is only represented by a common introduced weed. *Malvastrum* A. Gray is found in South Africa in the South-west and the Karroo and extends to Natal. *Sida* L., *Abutilon* Gaertn. and *Pavonia* L. radiate from the Tropics. I have no record of these genera west of the Uit. and P. E. divs. *Hibiscus* L. is generally distributed over South Africa. *Sphaeralcea* St. Hill. is confined to South Africa. It is found in the South-west, and then again in more eastern parts (Norval's Pont, Herschel district, Tembuland). *Althaea* L. is only known from near the mouth of the Orange River. The remaining five genera skirt the Tropics, *Cienfugosia* Cav. reaching Griqualand East and *Thespesia* Corr. Natal.

Sterculiaceae.

Sterculia L. is represented by one species near Uitenhage, famous on account of its complete isolation; the nearest spot where another species is found is the Lydenburg district, Transvaal. *Hermannia* L. (including *Makernia* L.) is generally distributed in South Africa. *Melhania* Forsk. and *Dombeya* Forsk. have their western limit in the Uit. dis. *Cola* Schott. is found in Pondoland and Natal and *Waltheria* L. in Natal and the Transvaal.

PARIETALES.**Ochnaceae.**

Ochna L. is found in the forest regions from the Knysna to Natal and the Transvaal. No South African locality for *Sauvagesia* L., said to occur in temperate South Africa, is known to me. The remarkable endemic genus *Roridula* L. is restricted to a circumscribed area in South-west Cape Colony (mainly in the Caledon div.).

Guttiferae (incl. Hypericineae).

Hypericum L. is found from George eastwards to Natal and the Transvaal. The only other South African genus, *Garcinia* L., grows from the Transkei eastwards.

Elatinaceae.

Bergia L. occurs on the Cape Peninsula, at Riversdale, in the Uit. div. and then again in Bechuanaland, the Transvaal, and Portuguese East Africa.

Frankeniaceae.

Only the genus *Frankenia* L. is found in South Africa. It occurs on saline soil in the West, South-west, the Karroo, and near Kimberley, but in the coast districts it is not recorded east of Uitenhage.

Tamaricaceae.

Tamarix L. occurs in Namaqualand.

Violaceae.

Hybanthus Jacq. (*Ionidium* Vent.) found in the southern coast districts has approximately its western limit in the Uit. div. *Viola* L., apart from garden escapes, is only found in South-West Cape Colony and the Transvaal, while *Rinorea* Aubl. (*Alsodeia* Thouars.) penetrates from the Tropics into Natal and Pondoland.

Flacourtiaceae.

Kiggelaria L. and *Scolopia* Schreb. extend in the coast districts from the Cape Peninsula to the Tropics, *Doryalis* Arn. et. Mey. (incl. *Aberia* Hochst.) and *Trimeria* Harv. from George to the Tropics. Seven other genera penetrate into South Africa from the Tropics, but only *Xylosma* G. Forst. goes very far, namely to the Pirie forest, and *Rawsonia*, Harv. et. Sond. to the Transkei.

Turneraceae.

Wormskioldia Schum. et Thom. and *Piriqueta* Aubl. (formerly under *Turnera*) are found in South Africa in the parts skirting the Tropics. Both are found in the Transvaal but do not reach Natal.

Passifloraceae.

None of the South African genera are found wild in the Uit. and P. E. divs. They are only found in the parts of South Africa adjoining the Tropics. (See the next order, *Achariaceae*.)

Achariaceae (endemic in South Africa).

Ceratocycos Nees and *Acharia* Thunb. are recorded from the Uit. and

P. E. divs. The former extends from Port Elizabeth eastwards to Natal, the latter from Van Stadens to the Transkei. The third genus, *Guthriea* Bolus, is absent. It is only known from the Oudeberg near Graaff Reinet.

Loasaceae.

This order is represented in Africa only by the endemic genus *Kissenia* R. Br. found in the Western Region.

Begoniaceae.

Begonia L. is found in the East and North-East, extending to the mouth of the Kei River.

OPUNTIALES.

Cactaceae.

The only native genus, *Rhipsalis* Gaertn., extends from the mouth of the S. John's River towards the Tropics. Several species of *Opuntia* Haw. are naturalised and are abundant in the south-eastern parts of the central region.

MYRTIFLORAE.

Geissolomaceae (endemic in South Africa).

Geissoloma Lindl. et Kunth, the only genus, which, moreover, is monotypic, occurs only in the Swellendam and Riversdale divs.

Penaeaceae (endemic in South Africa).

The occurrence of the *Penaeaceae* with five species of *Penaea* L. in the Uit. and P. E. divs. is of great interest as they are restricted to the "south-west coast region." None of them are found east of the Uit. div. The four other genera are not found east of Swellendam.

Oliniaceae.

The only genus, *Olinia* Thunb., is represented in the Uit. and P. E. divs. by the widely-spread, very variable species which has served as the type for the genus. It extends from the Cape Peninsula through the coast districts to Natal, the Transvaal, and beyond.

Thymelaeaceae.

Arthrosolen C. A. Mey., *Passerina* L. (endemic), *Gnidia* L. and *Lasiosiphon* Fresen. have fairly general distribution in South Africa; the last is absent from the western region and the Cape Peninsula. *Chrymococca* Meisn. (endemic) is found at Simons Bay and in the Van Rhynsdorp div., *Cryptadenia* Meisn. (endemic) in the South-west, extending to the Cold Bokkeveld, but not reaching Swellendam, *Lachnaea* L. (endemic) in the South-west extending to the Cold Bokkeveld and eastwards to the P. E. div.,

Struthiola L. (endemic) in the South-west extending to the Ceres div. and along the southern coast belt to the Albany div. and appearing further in East Pondoland. *Dais* L. is found in the North-east and East southwards to Kingwilliamstown (outside South Africa it only occurs in Madagascar). *Englerodaphne* Gilg. is recorded from East London to Natal, *Synaptolepis* Oliv. from Delago Bay, *Peddiea* Harv. from the North-east and East (extends to Komgha).

Lythraceae.

This order radiates from tropical Africa into South Africa. Of the six South African genera only *Lythrum* L. gets anywhere near to the south-west coast region, and this only by a widely spread species which is recorded from the Uit. and P. E. divs.

Rhizophoraceae.

Rhizophora L. and *Bruquiera* Lam. occur on the coast from the Transkei northwards; *Weihea* Spreng. in the coastwards forests of the East, extending to the Amatolas near Kingwilliamstown; *Cassipourea* Aubl. in East Pondoland and Natal.

Combretaceae.

Only *Combretum* L. reaches our districts from the East in one species which does not seem to extend beyond Van Stadens. *Quisqualis* L. and *Terminalia* L. skirt the Tropics, but do not reach Natal.

Myrtaceae.

Two species of *Eugenia* L. are recorded from the Uit. and P.E. divs. This genus does not seem to occur west of Van Stadens. *Metrosideros* R. Br. (south-western), *Syzgium* Gaertn., and *Heteropyxis* Harv. (both eastern) are absent.

Melastomaceae.

This order penetrates from the Tropics: *Memecylon* L. to Natal, *Dissotis* Benth. (incl. *Argyrella* Naud.) and *Osbeckia* L. to East Pondoland.

Oenotheraceae.

Only the genera *Oenothera* Spach. and *Epilobium* L. are recorded from the Uit. and P. E. divs. The former is introduced. The remaining three South African genera are eastern and are found in Natal and beyond, only *Ludwigia* L. comes as close as Kingwilliamstown. (*Montinia*, see under Saxifragaceae.)

Halorrhagidaceae.

Gunnera L., *Laurembergia* Berg. (*Serpicula* L.), and *Myriophyllum* L. are recorded from the Uit. and P.E. divs. The last does not seem to occur

west of the Uit. div., and does not occur in the Albany and Bathurst divs. The two former are widely spread in the coast districts of South Africa. *Gunnera* also occurs in the North-East, and *Myriophyllum* in the system of the Orange River.

UMBELLIFLORAE.

Araliaceae.

The only South African genus, *Cussonia* Thunb., is recorded from the Uit. and P. E. divs. It is very widely spread though thinning out westwards, only one species reaching the Cape Peninsula.

Umbelliferae.

The majority of South African genera have been recorded from the Uit. and P. E. divs. The exceptions (leaving out obvious garden escapes) are *Sanicula* L. and *Ptychotis* Koch (excl. *Trachyspermum* Link.), both with wide distribution east and west of Uitenhage; *Oenanthe* L., *Levisticum* Koch, and *Ruthea* Bolle (*Glia* Sond.), all found only in the south-western coast region; *Choritaenia* Benth. et Hook (*Pappea* Sond. et Harv.), with northern distribution; *Polemannia* E. and Z., with northern and eastern distribution as far as the Katberg. Of the recorded ones, *Hermas* L., *Cupnophyllum* Gaertn., and *Trachyspermum* Koch are south-western and do not extend in an eastern direction, while *Arctopus* L. and *Annesorhiza* Clam. et Schl. (incl. *Stenosemis* E. Mey.) with similar distribution reach the Albany district. *Pimpinella* L. with eastern distribution does not go further west, while *Heteromorpha* Ch. et Schl. extends to George. *Rhyticarpus* Sond., with northern distribution, is also found north of Grahamstown. *Pituranthos* Viv. (*Deverra* DC.), with central, northern, and western distribution, reaches neither Natal nor the Cape Peninsula. The other genera have more or less general distribution in the southern and south-eastern coast belt.

Cornaceae.

The genus *Curtisia* Ait. is reported from the Zuurberg. It is found in all forest regions in South Africa, extending even to the Cape Peninsula. The genus *Cornus* L. is recorded from Natal, and does not extend westwards.

METACHLAMYDEAE (Sympetalae).

ERICALES.

Ericaceae.

Erica L. is represented in the Uit. and P. E. divs. by twenty-eight species, many of which are only recorded from Van Stadens. East of Uit. div. the number is very much smaller. *Blaeria* L., *Thoracosperma* Kl., *Anomalan-*

thus Kl., and *Simocheilus* Kl., which are distributed in the south-western coast region, have their eastern limit in our districts. Fifteen genera found in the south-western coast region are absent. *Coilostigma* Kl. and *Thamnus* Kl. are recorded. The former extends from Riversdale div. to the Albany div.; the latter is only known from Van Stadens and the Humansdorp div. *Ericinella* Klotzsch is only known from the Graaff Reinet, Cathcart, and Queenstown divs. *Vaccinium* L. (frequently placed in a separate order, *Vacciniaceae*) is absent. In South Africa it is only known from Barberton.

PRIMULALES.

Myrsinaceae.

Myrsine L. is widely spread in the coast districts of South Africa. *Maesa* Forsk. extends from the Stockenström div. eastwards, and *Embelia* Burm. eastwards from the Komgha div.

Primulaceae.

Lysimachia L. is not found west of Uitenhage, and is even further east very local in its distribution. *Samolus* L. is found all over South Africa, where it has suitable conditions for its growth. *Anagallis* L. is absent from the Karroo and the western region. The only species recorded occurs in South Africa as a garden weed only.

Plumbaginaceae.

Statice Willd. is found in the western and south-western region, penetrating only slightly into the Karroo. Along the coast it is found as far as the Kentani district. It is absent from Natal. *Plumbago* L. has its western limit in the Uit. div., while *Vogelia* Lam. is, in South Africa, chiefly confined to the western region penetrating into Calvinia.

EBENALES.

Sapotaceae.

Sideroxylon L. is found in the coast districts from the Cape Peninsula eastwards to Delagoa Bay and the Transvaal. *Chrysophyllum* L. is confined to the Transvaal and Natal. *Mimusops* L. extends from the Transvaal and Natal along the coast to the Olifantshoek close to the eastern border of the Uit. div.

Ebenaceae.

Royena L. and *Euclea* Murr. are generally distributed. *Maba* J. R. and G. Forst. occurs on the coast from East London to Natal. *Diospyros* L. is only found on the borders of the Tropics.

CONTORTAE.

Oleaceae.

Jasminum L. is widely spread in the southern coast districts, and extends to the Orange Free State and the Transvaal, but does not reach the Cape Peninsula in the West. *Olea* L. has a similar distribution, but extends further into the interior, and is found on the Cape Peninsula. *Schrebera* Rosb. is confined to the Transvaal; while *Menodora* Humb. et Bonpl. is found in Little Namaqualand, Griqualand West, Bechuanaland, the Transvaal and Natal.

alvadoraceae.

Azima Lam. is widely spread in the southern coast districts of South Africa, but does not reach the Cape Peninsula. The only other South African genus, *Salvadora* Garcin., enters South Africa only near Delagoa Bay.

Loganiaceae.

Gomphostigma Turcz. has only been found in the Uit. div. on its eastern boundary. It is absent from the coast districts westwards; otherwise it is widely spread in South Africa, especially in the eastern and north-eastern parts. *Nuzia* Lam. is absent from the Karroo and the coast districts west of George; otherwise it is also widely spread. *Chilanthus* Burch. and *Buddleia* L. are more widely spread, but absent from the Cape Peninsula. *Strychnos* L. has its western limit in our area, and extends in the coast districts to Delagoa Bay and is also found in the Transvaal. *Retsia* Thunb., placed by some botanists under *Solanaceae*, is only found in a restricted area in South-West Cape Colony.

Gentianaceae.

The large genera *Sebaea* R. Br. and *Chironia* L. are practically absent from the Karroo; otherwise they are generally spread over South Africa. The monotypic genus *Lagenias* R. Br., which is sometimes kept separate from *Sebaea*, is confined to West and South-West Cape Colony. *Belmontia* E. Mey. (incl. *Ezochaenium* Griseb.) is confined to the Kalahari and the south-eastern region from the Kei mouth onwards. *Orphium* L. is only found in South-West Cape Colony as far as Riversdale. *Enicostemma* Blume is restricted to the North-East, skirting the Tropics. *Faroe* Welw. is only found in Basutoland; *Suertia* L. in the Orange Free State and the Transvaal. Then there are two genera confined to marshy ground. *Villarsia* Vent. grows in the coast districts from the Cape Peninsula to the Uit. div., while *Limnanthemum* S. M. Gmel. extends from the Cape Peninsula through the coast districts to Natal and the Transvaal.

Apocynaceae.

Of the thirteen genera known from South Africa, *Gonioma* E. Mey. is

entirely restricted to forests from George to East London. All the others are outliers from the Tropics. *Carissa* L. extends westwards to Swellendam; *Acokanthera* G. Don. to George. They are absent from the Karroo. *Pachypodium* Lindl. extends into the Eastern Karroo, and in the coast districts is not recorded west of Uitenhage. The remaining genera do not occur west of the Kei River, with the exception of *Strophanthus* DC., which extends to the Kagaberg, near Bedford.

Asclepiadaceae.

(1) Periploceae. *Ectadium* E. Mey. is endemic in Namaqualand. None of the other five South African genera extend further from the Tropics than the Kei River, with the exception of *Raphionacme* Harv., which is found as far as the Uit. div.

(2) Secamoneae. *Secamone* R. Br. extends from the Tropics in the coast districts; one species reaches the Cape Peninsula.

(3) Cynancheae. Of these only *Cynanchum* L. and *Stapelia* L. are found in all South African regions and extend into the Tropics, *Huernia* R. Br. in all except the extreme S.W. No less than eighteen genera are endemic in South Africa, namely, *Microlooma* R. Br. (W., S.W. to Port Elizabeth, Karroo and Bechuanaland), *Parapodium* E. Mey. (Kalahari region to Eastern Karroo and Queenstown), *Woodia* Schlecht (E. and N.E. to Somerset East and Humansdorp), *Periglossum* Decne. (E. and N.E. to Komgha), *Cordylogyne* R. Br. (N.E. and E. to Graaff Reinet and Queenstown), *Krebsia* Harv. (N.E. and E. to Aliwal North and Fort Beaufort), *Fanninia* (Natal to Cathcart), *Glossostephanus* E. Mey. (S.W. to Van Stadens), *Eustegia* R. Br. (W. and S.W. to the Paarl), *Emicocarpus* K. Schum. & Schlecht. (Delagoa Bay), *Emplectanthus* N. E. Br. (Natal and Zululand), *Rhysolobium* E. Mey. (Little Namaqualand), *Sisyranthus* E. Mey. (Natal, Griqualand East to the Winterberg, Albany and Bathurst), *Macropetalum* Burch. (Kalahari region), *Anisotoma* Fenzl. (Drakensberg to Van Stadens), *Pectinaria* Haw. (West and Karroo to Bedford), *Diplocyatha* N. E. Br. (Western Karroo), *Piarranthus* R. Br. (West and Karroo to Bedford). The remaining genera are represented in the Tropics. Their distribution in South Africa is as follows: *Xysmalobium* R. Br. (in all South African regions except the West, Cape Peninsula, Western Karroo), *Schizoglossum* E. Mey. and *Asclepias* L. (mostly E. and N.E., penetrating into the Eastern Karroo and along the coast to the Cape Peninsula), *Pachycarpus* E. Mey. (mostly E. and N.E., extending in the coast districts to Swellendam), *Pentarrhinum* E. Mey. (fairly general, but in the southern coast districts, not west of Uit. div.), *Sarcostemma* R. Br. (similar, but not west of Mossel Bay), *Pergularia* L. (N. and N.E.), *Tylophora* R. Br. (W., N., N.E. and E., not west of Uit. div. in the southern coast districts, penetrating slightly into the Karroo), *Sphaerocodon* Benth. (E. and N.E.),

Marsdenia R. Br. (E., penetrating westwards to the Cradock and Uit. div.), *Fockea* Endl. (Karoo, E. and N.E., one species in the South as far as Mossel Bay), *Gymnema* R. Br. (E., N. and N.E.), *Orthanthera* Wright (W. and Kalahari region), *Tenaris* E. Mey. (N.E., one species in Albany, Bathurst and at Addo), *Riocreuxia* DC. (N., N.E. and E., extending westwards to the Karroo and George, but not recorded from our area), *Ceropegia* L. (N., N.E. and E. to the Uit. div. and Eastern Karroo), *Brachystelma* R. Br. (mainly N.E. and E. to Uit. div., few in Eastern Karroo, W. and S.W.), *Caralluma* R. Br. (W., as far as Malmesbury and Ceres, Western Karroo, Kalahari region), *Trichocaulon* N. E. Br. and *Hoodia* Sweet (W. and Karroo), *Tavaresia* Welw. (W. and Upper Karroo), *Huerniopsis* N. E. Br. (Griqualand East and Bechuanaland), *Duvalia* Haw. (W. and Karroo to Albany and Uit. div.).

TUBIFLORAE.

Convolvulaceae.

Ipomaea L. is mainly north-eastern and eastern, extending westwards in the coast districts, but only few species are found west of the Uit. div., none reaching the Cape Peninsula. *Convolvulus* L. is found in all South African regions, but is almost absent in the West, and there is no native species on the Cape Peninsula. *Falkia* L. is found in the southern coast regions, also in the East and North-East. *Cuscuta* L. has a similar distribution. *Dichondra* Forst. is found in the coast districts from the Cape Peninsula to Natal. Then there are four genera only found in the East and North-East, of these *Breweria* R. Br. extends to Queenstown, and lastly one genus is only recorded from Natal and Delagoa Bay.

Hydrophyllaceae.

The only South African genus, *Codon* L., is found in the western region extending to the Ceres div.

Borraginaceae.

The following are found in all regions with the exceptions noted *Heliotropium* L. (absent from south-east coast region), *Cynoglossum* L. (absent from Western Karroo and western region), *Echinosperrum* L. (absent from West and extreme South-West), *Myosotis* L. (absent from West, including Cape Peninsula), *Lithospermum* L. (absent from Cape Peninsula, except an evidently introduced species). *Cordia* L. is eastern, but extends to the Uit. div. and a considerable distance into the Karroo. *Tournefortia* L. has central, western, and northern distribution. *Trichodesma* R. Br. has northern, north-eastern, central, and western distribution, extending to the Ceres division. *Tysonia* Bolus is confined to Griqualand East. *Anchusa* L. is found in the Karroo (extending to Queenstown and neighbourhood of

Grahamstown) and in the West and South-West (not on the Cape Peninsula), *Lobostemon* Lehm. is mostly confined to the South-West, but also found in the West and the Karroo (extending to the Uit. and Albany divs.). The only native species of *Echium* L. is found in the Stellenbosch div.

Verbenaceae.

Bouchea Cham. is represented in all South African regions. Four (or five) genera are endemic in the south-west coast region, of these only *Stilbe* Berg. extends to the Uit. div. The remaining genera have a decided eastern and north-eastern distribution. Of these *Lippia* L. extends westwards to Grahamstown, *Priva* Adans. to our area, *Vitex* L. to Komgha, *Clerodendron* L. to Port Alfred, *Avicennia* L. to Peddie. *Duranta* L. (at least in the native state) does not come near our area, *Lantana* L. is absent in the West and South-West (except one introduced species). The species of *Verbena* L. found in various parts of South Africa are all introduced.

Labiatae.

Genera of more or less general distribution in South Africa: *Mentha* L., *Salvia* L., *Stachys* L., *Acrotoma* Benth. (absent from southern coast districts), *Leonotis* R. Br., and *Teucrium* L. (absent from the West). Sixteen genera have an essentially eastern and north-eastern distribution. Of these the following penetrate, mostly in the coast districts, a variable distance westwards: *Becium* Lindl. (to Uit. div.), *Ocimum* L. (to Komgha), *Orthosiphon* Benth. (to the Pirie bush or even to Albany [?]), *Syncolostemon* E. Mey. (to East London), *Plectranthus* L'Herit. (to Caledon), *Coleus* Lour. (to the Peddie div.), *Aeolanthus* Mart. (to Graaff Reinet), *Micromeria* Benth. (to Queenstown), *Leucas* Burm. (to the Uit. div.), *Lasiocorys* Benth. (to the Uit. and Jansenville divs.), *Ajuga* L. (to the Uit. and Somerset East divs.). Two genera are only found in the north-eastern parts, *Ballota* L. in the western, south-western, and central parts (to Queenstown). *Cedronella* Moench. (South-west) and *Marrubium* L. (Table Mt. and Addo) are introduced.

Solanaceae.

Solanum L., *Withania* Pauq., and *Lycium* L. are found in all South African regions, the last especially in the drier parts, *Physalis* L. (with the exception of one Natal species?), *Nicandra* Adans., *Datura* L., *Nicotiana* L., and *Cestrum* L., are introduced, but have become naturalised in numerous places. (For *Retzia* Thunb., see under *Loganiaceae*.)

Scrophulariaceae.

Twenty-one genera have a very wide distribution in South Africa. Of these only *Teedia* Burch. and *Bellardia* All. are not recorded from the Uit. and P. E. divs., *Mimulus* L. and *Striga* Lour. (common in th

Albany distr.), are absent from the south-west coast region, *Polycarena* Benth., *Peliostomum* Endl., *Hemimeris* L. f. are not found east of Uitenhage in the coast regions, *Hyobanche* Thunb. is absent in the Kalahari region, *Ilysanthes* Rafin. is absent from the central and western region. Seven genera have eastern distribution and do not reach our districts. The four following are mainly eastern and north-eastern: *Sopubia* Hamilt. (to the Knysna), *Bopusia* Presl. (to Riversdale), *Buchnera* L. (to the Paarl), *Cynium* E. Mey. (extends west of Uitenhage), *Ramphicarpa* Benth. (to George). Three genera are south-western, of which *Freylinia* Spin. finds its eastern limit near Port Elizabeth, Three genera are only found in the Kalahari region, two only in the western region. There remain only the following (leaving out three genera which must be looked upon as garden escapes): *Aptosimum* Burch. (recorded from the foot of the Winterhoek Mts., central and western regions and Kalahari) and *Alonsoa* R. & P. which has only so far been found by Zeyher near Uitenhage.

Tribe Selagineae.

Hebenstreitia L., *Walafrida* E. Mey. (poorly developed in the West and South-West) and *Selago* L. (mostly in the South-West) are found in all South African regions. *Dischisma* Choisy is confined to the West and South-West, extending to Port Elizabeth, *Microdon* Choisy, *Gosela* Choisy, and *Agathelpis* Choisy to the South-West.

Bignoniaceae.

Tecomaria Spach extends from Uitenhage to the Tropics in the south-eastern coast belt. *Podranea* Sprague is confined to Pondoland. *Rhigozum* Burch. extends from the northern parts through the Karroo to the Keiskamma, Uitenhage, and George. *Catophractes* D. Don has western and north-eastern distribution. *Kigelia* DC. extends from Delagoa Bay to the Pirie forest in rather isolated spots.

Pedalinaceae.

This order has chiefly northern and north-eastern distribution in the so-called Kalahari region. Only *Ceratotheca* Endl. and *Sesamum* L. extend to Natal and the former also to Pondoland, while the latter also extends to the Richmond and Somerset East divs. The endemic genus *Holubia* Oliv. occurs in the Transvaal, *Rogeria* J. Gray inhabits the western region and extends to the Kalahari region. The southern limits of the remaining genera are as follows: *Pterodiscus* Hook. and *Pretrea* J. Gay extend to Griqualand West, *Harpagophytum* DC. to Hopetown.

Orobanchaceae.

Only the genus *Orobanche* Tournef. (including *Philipaea* E. Mey.) is

found in South Africa and represented in the south-west coast region. It has not been recorded from the Uit. and P. E. divs.

Gesneriaceae.

The only South African genus, *Streptocarpus* Lindl., is mainly distributed in the eastern and north-eastern parts, and extends in the coast districts to the Knysna.

Lentibulariaceae.

Utricularia L. is found in the East and North-east extending in the coast districts to the Cape Peninsula and inland to Graaff Reinet. The only other South African genus, *Genlisea*, A. St. Hil. is only found in the East and North-East.

Acanthaceae.

Blepharis Juss. and *Monechma* Hochst. are found in all South African regions except in the extreme South-West; *Barleria* L. has a similar distribution, but is also absent from the West Karroo. Ten genera come across the border of the Tropics but do not penetrate very far in South Africa. One genus is endemic in the Kalahari, and one, *Chaetacanthus* N. ab E., is endemic in the north-eastern and eastern parts but extends in the coast districts to Mossel Bay. The following also extend from the Tropics into eastern and south-eastern parts of South Africa, but have a considerable western extension in the coast districts: *Thunbergia* L. (one species to Mossel Bay), *Ruellia* L. (one to Swellendam) *Sclerochiton* Harv. (one to Somerset East), *Crabbea* Harv. (extends to the Uit. div.), *Asystasia* Blume (one said to come from Koonap heights near Grahamstown and one has been found at Ceres), *Justicia* L. (few go west of Uitenhage), *Siphonoglossa* Oerst. (to Alexandria div.), *Isoglossa* Vent. and *Peristrophe* Nees (not recorded west of the Uit. div.), *Hypoestes* R. Br. (to Caledon), *Dicliptera* Juss. (to Cape Peninsula).

Myoporaceae.

The endemic genus *Optia* Adans. occurs in the western and south-western region as far as Riversdale.

PLANTAGINALES.

Plantaginaceae.

Only the genus *Plantago* L. is found in South Africa. It occurs in the southern coast districts (penetrating to Graaff Reinet) and in the eastern and north-eastern parts.

RUBIALES.

Rubiaceae.

Seven genera are only found on the borders of the Tropics. There are

eleven genera found in the eastern and north-eastern parts which do not go west of the Kei: *Alberta* (to Pondoland), *Cephalantha* L., *Pachystigma* Hochst. (*Fadogia* Schweinf.), *Richardsonia* L., *Mitracarpus* Zucc., *Tarenna* Gaertn. (*Webera* Schreb.), *Ozyanthus* DC. (to Port St. John's), *Tricalysia* A. Rich. (incl. *Bunburya* Meisn., *Diplospora* DC., and *Kraussia* Harv. to Pondoland), *Pentas* Benth. The following are also eastern and north-eastern but go much further west: *Pentania* Harv. (to the Winterberg and Port Alfred), *Plectronia* L. (*Canthium* Lam., to the Cape Peninsula). *Vangueria* Juss. (to the Uit. div.), *Pavetta* L. (to Port Elizabeth), *Rubia* L. (to the Gamtoos River valley, Oudtshoorn, and Boschberg), *Hydrophylax* L. (to the Zitzikamma), *Barresia* G. W. Mey. (sub-*Spermacoce* Dill., to Grahamstown), *Galopina* Thunb. (to Swellendam), *Psychotria* L. (to the Uit. div.), *Gardenia* E. Mey. (to Swellendam), *Burchellia* R. Br. (to Swellendam), *Randia* Hochst. (*Stylocoryne*, Cavan., *Mitrostigma* Hochst., to the Uit. div.). The following have more restricted distribution: *Carpocoe* Sond. (Cape Peninsula to Uitenhage and Grahamstown), *Nenax* Gaertn. (*Ambraria* Cruse, South-West, and again one species at Port Alfred and East London), *Crocylis* E. Mey. (Namaqualand). Fairly generally distributed are only *Galium* Tourn., *Anthospermum* L., and *Oldenlandia* Plum. (incl. *Hedyotis* L.).

The following genera are endemic in South Africa: *Carpocoe* Sond., *Nenax* Gaertn., *Galopina* Thunb., *Crocylis* E. Mey., and *Burchellia*, R. Br.

Valerianaceae.

Valeriana L. is found in the coast districts from the Cape Peninsula to Natal, also on the eastern mountains. *Valerianella* Poll. appears to be only known from Zuurbrak and the Grootvadersbosch.

Dipsacaceae.

Cephalaria Schrad. and *Scabiosa* L. are eastern and north-eastern and extend in the coast districts to the Cape Peninsula.

CAMPANULATAE.

Cucurbitaceae.

Of the 17 South African genera most are found in the parts adjoining the Tropics. The genera *Melothria* Cogn. (incl. *Mukia*, Arn. and *Zehneria* Endl.), *Coccinia* Wight et Arn. (*Cephalandra* Schrad.), *Cucumis* L., and *Kedrostis* Cogn. (incl. *Coniandra* and *Rhynchocharpa*) are recorded from the Uit. and P. E. divs. Few species are found west of the Uit. div., only 3 or 4 reaching the Cape Peninsula.

Campanulaceae.

Grammatotheca Presl. (endemic), *Monopsis* Salisb. (incl. *Dobrowskya* Presl. and *Parastranthus* Don), *Lobelia* L. (incl. *Isolobus* DC. and *Metzleria*

Presl), *Cyphia* Berg., *Wahlenbergia* Schrad. and *Lightfootia* L'Her. are common in the coastal belt from Clanwilliam to Natal and in the eastern mountains, but are poorly represented in the Karroo and the Kalahari region proper. *Laurentia* Neck. (incl. *Enchysia* Presl.) is chiefly found in the South-West, but one species occurs near Grahamstown and one still further east. *Prismatocarpus* L'Her. (endemic) is essentially south-western, but extends to Grahamstown. *Roëlla* L. (endemic), also mostly south-western, occurs as far as the Uit. div. and then again from East London to Natal. The following south-western genera (all endemic) do not reach the Uit. and P. E. divs.: *Microcodon* A. DC., *Siphocodon* Turcz., *Rhigiophyllum* Hochst., *Merciera* A. DC. and *Treichelia* Vatke (*Leptocodon* Sond.). *Cephalostigma* A. D.C. enters temperate South Africa in the extreme North-West, *Sphenoclea* Gaertn. in the North East.

Goodeniaceae.

The only South African genus, *Scaevola* L., represented by a single species inhabiting the sand on the seashore, is recorded from the Uit. and P. E. divs. and extends both East and West, in the latter direction as far as Mossel Bay.

Compositae.

The enormous number of South African genera of *Compositae* can be arranged in several groups more or less well defined as regards their distribution.

1. Over 40 genera with more or less general distribution (some restricted to the southern coast districts and some not reaching to the Cape Peninsula). Of these *Dicrocephala* DC. and *Chrysanthemum* L. are not found in the Uit. and P. E. divs.

2. About 20 genera, radiating from the Transvaal and Natal chiefly along the southern coast districts. Most of these do not come near our districts. *Tanacetum* L. stops at Bedford and East London, *Schistostephium* L. in the Albany district. *Artemisia* L., which extends to Genadendal, is not recorded. *Vernonia* Schreb. and *Stephanocoma* Less. extend much further west, while *Mikania* Willd. has its western limit at Van Stadens and *Launea* Cass. (*Microhynchus* Less.) at Coega.

3. Over 40 genera are mainly found in the south-west coast region, some with very local distribution. Of these about 28 do not reach our districts. *Oedera* L. and *Landtia* L. are not recorded though occurring further east, namely near Grahamstown. Amongst the recorded ones *Corymbium* L. *Disparago* Gaertn., *Helipterum* DC., *Oldenburgia*, Less., reach the neighbourhood of Grahamstown. *Elytropappus* DC., though common near Grahams-town, is introduced, and probably also reached the Uitenhage district through the agency of man; *Heterolepis* Cass., *Cullumia*, R. Br., have their eastern limits in our districts, *Gorteria*, L. at Hankey.

4. Genera with essentially central and south-western distribution. There are about 9 of these, of which the following are recorded: *Chrysocoma* Cass., *Adenachaena* DC., *Eriocephalus* L., all of which extend to the neighbourhood of Grahamstown, while *Doria* Less. and *Didelta* L'Hér. find their eastern limit in our districts.

5. A few genera are found in the central region and in our districts namely, *Phymaspermum* Cass., *Amphiglossa* DC., and *Nestlera* Spreng.

6. Genera with very restricted distribution and not included in 2-5. Of these the following require mention: *Cypselodontia* DC. (only found by Ecklon in the Uitenhage district), *Anaglypha* DC. (only found by Drege between Coega and the Zwartkops river), *Platycarpha* Less. (Uitenhage and Natal), *Adenosolen* DC. (Uitenhage, very incompletely known; described from Ecklon's specimens), *Amphidoza* DC. (Uitenhage, Port Elizabeth, Grahamstown, Western Transvaal).

There remain 6 genera with western distribution, 3 genera confined to the central region and a few genera with very local distribution in other parts.

NOTES ON

- (a) RADIATION PATTERNS OF THE TRANSFORMATION OF
MAGNESIUM HYDROXIDE TO MAGNESIUM OXIDE.
- (b) SERPENTINE, MALACHITE, AND PSEUDOMORPH QUARTZ.
- (c) BULTFONTEIN APOPHYLLITE.
- (d) DIAMOND TESTS BY RADIATION PATTERNS.
- (e) DIFFRACTION FROM THE EDGES OF A SQUARE PLATE
OF IODINE.

By J. STEPH. V. D. LINGEN.

(With Plates I-VI.)

(a) RADIATION PATTERNS OF THE TRANSFORMATION OF MAGNESIUM
HYDROXIDE TO MAGNESIUM OXIDE.

In his lecture before the eighty-fifth Naturforscherversammlung in Vienna, Frederich described the appearance of certain Röntgen interference phenomena in the pattern of certain crystals.*

These phenomena consist of the usual pattern of points lying on conic sections, and in addition the phenomenon of radial lines emanating from these points towards the central point.

This latter phenomenon he ascribed to a two-dimensional grating effect of the crystal, co-existent with the usual three-dimensional grating.

Subsequently von Lane and the author contended that this phenomenon was not due principally to the space-lattice of the crystal, but rather to cleavage planes, which disturb the regularity of the three-dimensional grating.†

If the regularity is disturbed in one direction, only two of the three factors which determine the maximum intensity remain; hence the lattice of the crystal as a whole tends to become a two-dimensional one.

This latter contention is supported by the observations on the change of magnesium hydroxide to magnesium oxide.

On transmitting Röntgen rays parallel to the trigonal axis of a microscopically perfect piece of Brucite it showed the usual interference pattern of points without radial lines.

* 'Phys. Zeit.', 14, 1079, 1913.

† 'Nat. Wissenschaft,' 13, 319, 1914.

On heating the same crystal over 300 degrees, and obtaining its interference pattern at this temperature, the plate showed only six radial lines, with no trace whatsoever of the usual pattern.

The same specimen was then examined at room-temperature. In this case, *ceteribus paribus*, the plate showed an increase in the number of radial lines, but no interference points. That there are fewer radial lines in the former case than in the latter is due to the Debye effect.*

The intensity curve of the incident Röntgen rays is reflected in these radial lines, which now represent the complete reflected spectrum.

On examining the specimen macroscopically it was found to be now perfectly opaque, and so brittle that it crumbled into small flakes upon touching it.

The experiments recently carried out by the author show that the displacement of water molecules causes the displacement of the three principal points of the usual pattern, so that they become drawn out along radial lines, which seems to indicate a uniform displacement of the reflecting planes (*vide* Pl. I, Figs. 1 and 2 and Pl. II, Fig. 3).

Consequently we may conclude that whenever radial lines appear in the interference patterns there has been a weakening of the lattice of the original homogeneous crystal.

(b) SERPENTINE, MALACHITE, AND PSEUDOMORPH QUARTZ.

Serpentine.

Some crystallographers classify Serpentine as triclinic, whereas others simply state that the form of crystallisation is uncertain. Therefore it may be interesting to consider what light treatment with X rays may throw upon the matter. Moreover, it appears to be physically analogous with pseudoisotropic liquids when placed in a magnetic field.

It seems analogous only in so far as one axial direction is fixed.

A plate of Carolina (Transvaal) serpentine $\frac{1}{2}$ mm. thick was placed so that the threads were perpendicular to the incident rays. After an exposure of four hours the photographic plate showed a pattern symmetrical about the direction of the threads. The radial lines parallel to the threads showed a greater intensity than the subordinate lines, which are inclined at 30° and 60° respectively to the threads (*vide* Pl. II, Fig. 4).

On rotating this crystal round the direction of the threads through an angle of 60° , the subordinate lines were hardly discernible after an exposure of three hours.

If this mineral be heated the subordinate lines would probably vanish, and the pattern would consist only of a line parallel to the threads—phenomenon displayed by pseudoisotropic layers of para-azoxyanisol after an exposure of some forty hours.

* 'Phys. Zeit.' 15, 75, 1914; 'Nat. Wissenschaft,' 15, 371, 1914.

The pattern clearly indicates that the lattice of Serpentine has symmetry in a definite direction relative to the threads; hence it cannot be triclinic, which shows no symmetry whatsoever,* in interference patterns, except when it is uniformly twinned.

Malachite.

Secondly, a plate of Malachite, cut parallel to the slightly radiating fibres, was examined. After an exposure of three hours, the pattern showed three lines parallel to the threads and a number of secondary lines perpendicular to the principal line.

The two lines parallel to the principal line are faint in comparison with the principal line and the short secondary lines (*vide* Pl. III, Fig. 5).

Pseudomorph Quartz (Crocidolite).

A plate of Pseudomorph Quartz was examined, and the photogram, after an exposure of six hours, showed an intense dispersion round the central spot, accompanied by thin radial lines, making a small angle in the direction of the threads. This seems to indicate that this substance is microcrystalline and that the miniature crystals are so orientated that their normals to their reflecting planes favour the direction of the threads (*vide* Pl. III, Fig. 6).

Experiments on minerals of a similar nature are still in progress.

As far as these observations go, they seem to indicate that though the crystals show the characteristics of microcrystalline substances, nevertheless they also indicate that the elementary units still favour particular directions, and these directions give an index to the nature of the original lattice.

These radial lines indicate the loci of the principal points of the point-pattern of the original crystal, and also give an indication of the stability of the lattice.

Thus a new field of research is opened with regard to the transition stages in crystals. We wish to thank Prof. Young for supplying us with the specimens.

(c) BULTFONTEIN APOPHYLLITE.

In an earlier communication it was shown that a slight displacement of any cleavage planes within the crystal would cause an irregular distribution of the intensity in any individual spot to which that plane under ordinary conditions contributes its share of reflection, *i.e.* the intensity which it would have added to a definite part of the spot is now reflected to some other position, consequently irregularities caused by cleavage cracks will produce irregular spots.†

* Haga and Jaeger: 'Proc. Kon. Akad.,' Amsterdam, 15, 1552, 1916.

† 'Trans. Roy. Soc. South Africa,' vol. v, part 5, 571, 1916.

In order to test this, two specimens of Apophyllite were irradiated normally to their planes of cleavage. The first specimen showed a discontinuity along a plane of cleavage. This discontinuity did not extend throughout the crystal, and could only be detected microscopically after very careful examination. The second specimen, for which I am indebted to Prof. Shand, was perfect.

The interference pattern of the first showed a slight distortion of the points, which presented a nebular appearance (Pl. IV, Fig. 7), whereas the second specimen showed well-defined elliptic spots.

(d) DIAMOND TESTS BY RADIATION PATTERNS.

1. *Macle*.
2. "Spotted Stone."
3. Spotted Rejection Stone.
4. Inferior Brown Block with black spot in it.

1. *The Macle*,

which was examined some years ago, showed the usual pattern, together with a second one similar to the first, but rotated through 180° . This shows that the second lattice is similar to the first, but that it is rotated through two right angles. The spots in the pattern do not indicate any discontinuity in the lattice in the particular specimen (Pl. IV, Fig. 8).

2. "Spotted Stone."

This stone, apparently perfect but for a small black spot in the interior, showed after an exposure of three hours that some of the spots in the regular pattern were affected by the presence of the spot in the crystal. The slight discontinuity in some of the spots is only revealed when the photographic plate is placed at a considerable distance from the crystal.

The discontinuities in the various spots show that the disturbing influence is not the same for all the spots affected, but that the lattice of this stone as a whole remains perfect; that is to say, that there is neither a fracture in the lattice nor a rotation of its component parts (*vide* Pl. V, Fig. 9).

At this point we wish to express our indebtedness to the De Beers Consolidated Mines for the loan of this stone.

3. *Spotted Rejection Stone*.

This stone, bounded by a fairly smooth octahedral cleavage plane, was irradiated normally to it. After an exposure of three and a half hours the pattern showed that the lattice was ruptured, so much so that the usual elliptic spots were represented by irregular discontinuous markings, spaced at various intervals. The diamond as a whole seems to consist of units

more or less similarly orientated, each of which tends to produce its individual pattern.

The effect produced by these elements is augmented by rotating the crystal through a small angle, because the spots which recede from the centre become elongated, and hence show their component parts. In a perfect diamond the receding spot would be a uniformly drawn-out ellipse, whereas in the Rejection Stone it becomes a group of irregular spots (Pl. V, Fig. 10).

4. Inferior Brown Block with black spot in it.

This stone was also irradiated normally to an octahedral face. After an exposure of three hours the pattern showed that the crystal as a whole has less discontinuities in its lattice than the Rejection Stone. The pattern, however, shows that the distribution of the intensity in the spots is not uniform, and in addition there are several markings which are not caused by the regular lattice (Pl. VI, Fig. 11).

At this instance we wish to express our thanks to Dr. Percy A. Wagner for kindly lending us the above-mentioned stones.

(e) DIFFRACTION FROM THE EDGES OF A SQUARE PLATE OF IODINE.

A 4 mm. square plate of iodine was placed normal to the diaphragm in order to obtain its interference pattern. After an exposure of about one hour the experiment was discontinued, and the plate developed. This plate showed the unexpected phenomenon of diffraction from the edges of the crystal (Pl. VI, Fig. 12).

In some respects it is similar to the phenomenon described by Prof. Laub of Buenos Ayres.

These investigations were carried out by means of the grant in aid of Research, for which we wish to express our sincere thanks to the Government.

X-RAY ROOM,
SOUTH AFRICAN COLLEGE,
CAPE TOWN.

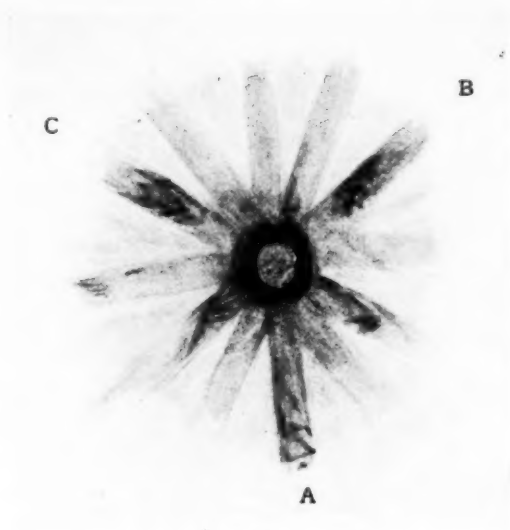


Fig. 1.

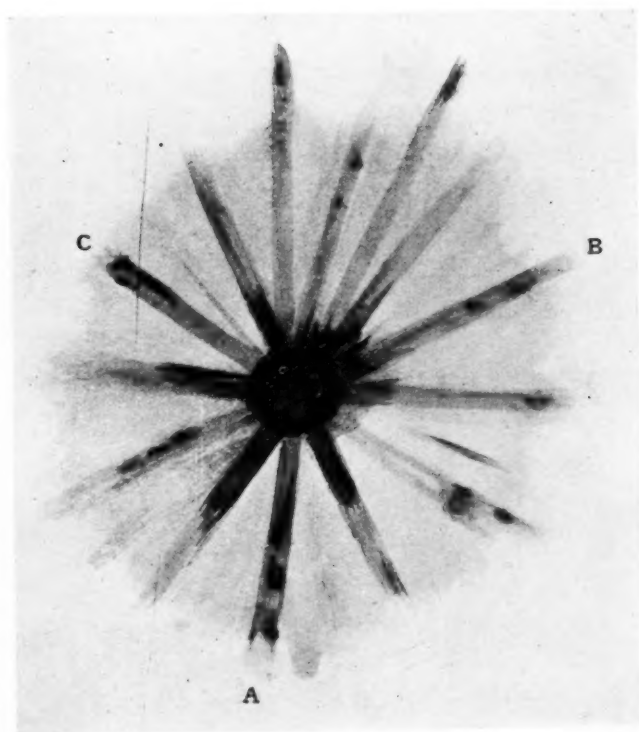


Fig. 2.

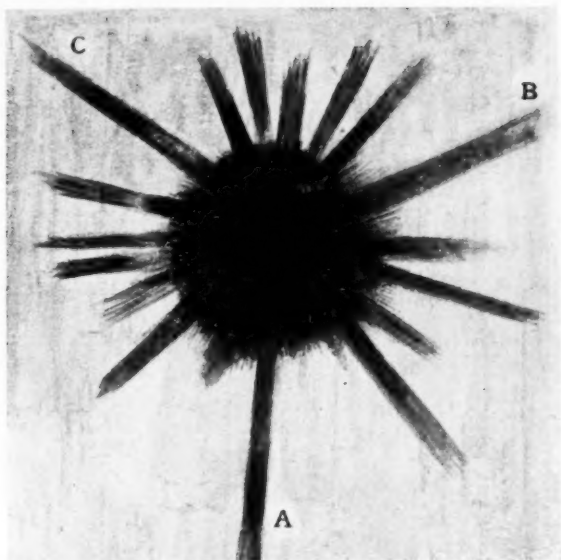


Fig. 3.

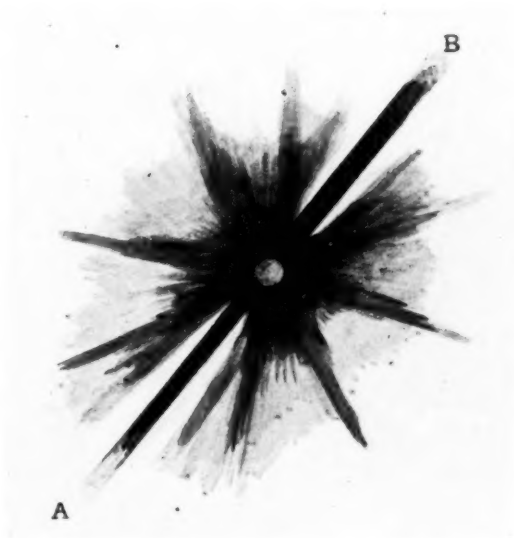
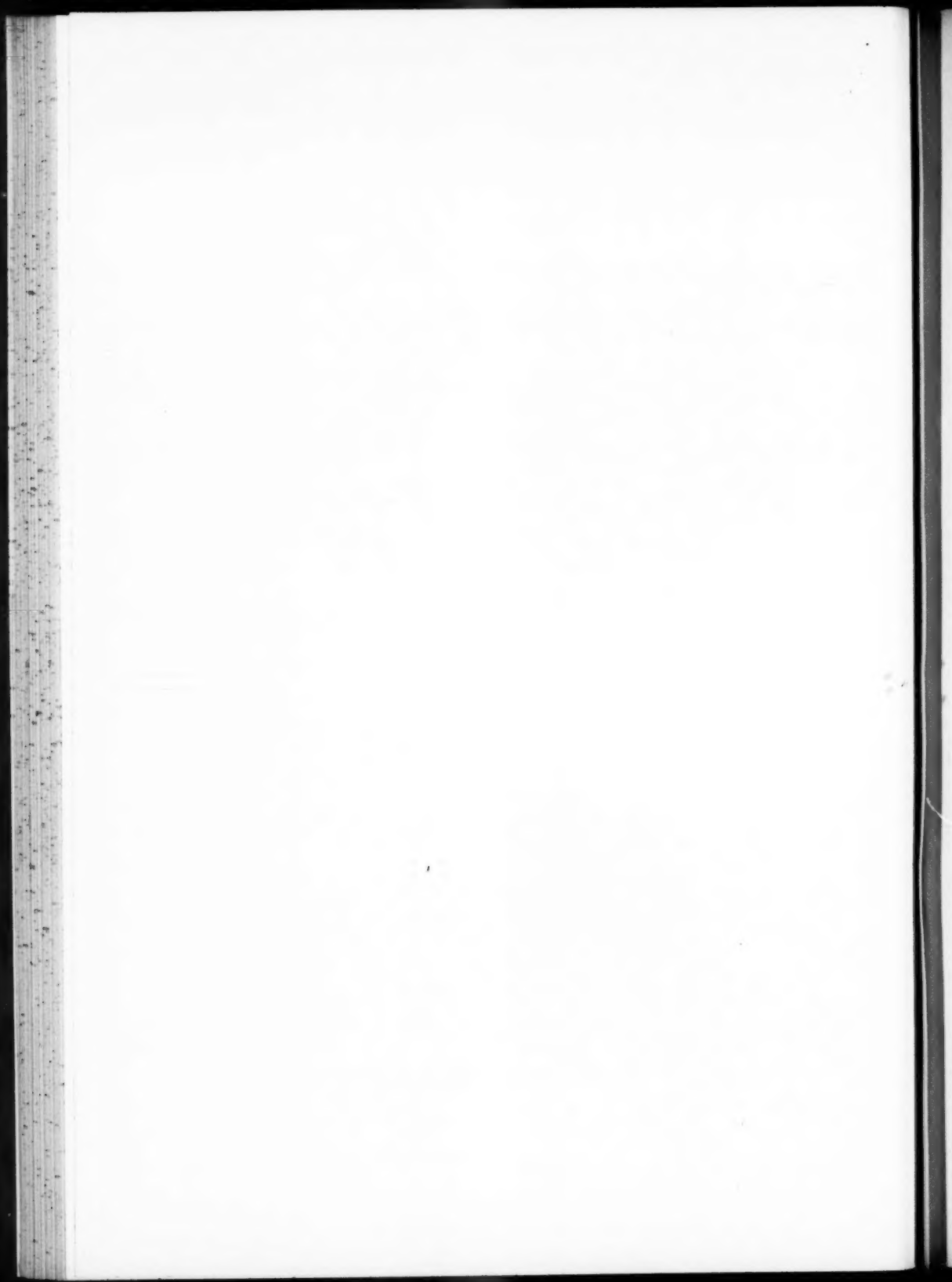


Fig. 4.



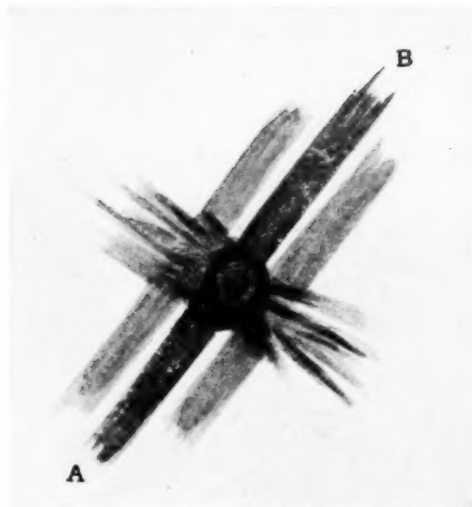


Fig. 5.

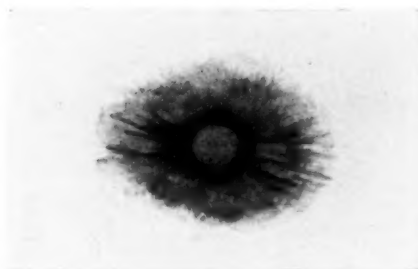


Fig. 6.



Fig. 7. Bultfontein Apophyllite.

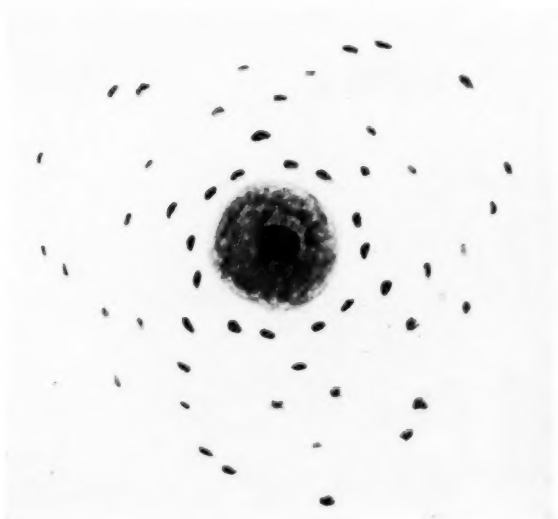


Fig. 8. Twin Diamond.

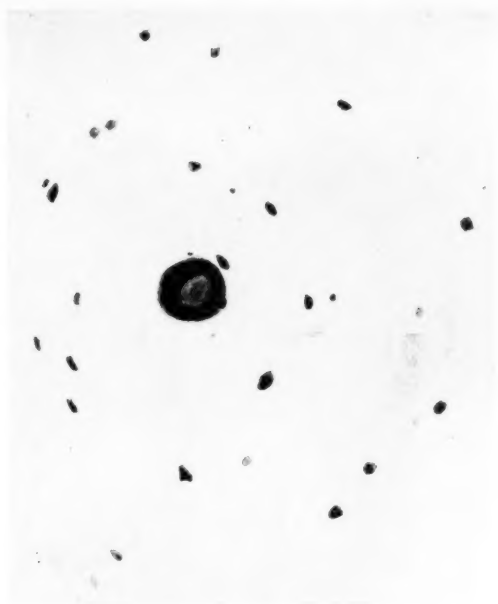


Fig. 9. Diamond Spotted Stone.



Fig. 10. Diamond Rejection Stone.



Fig. 11.

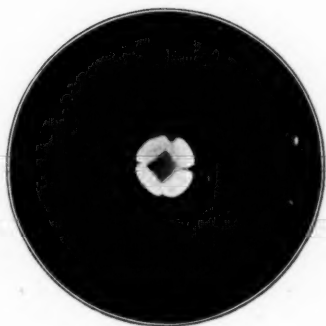


Fig. 12.

KIMBERLEY DIAMONDS: ESPECIALLY CLEAVAGE DIAMONDS.

By J. R. SUTTON, M.A., Sc.D., F.R.S.S.A.

1. LOCAL CHARACTERISTICS.

The difference in character between the diamonds obtained from the different Kimberley mines has been noted often enough; albeit not always in terms that convey much idea of their meaning to most people who are not directly employed in the diamond trade. Indeed, it is doubtful if mere words are competent to express the subtle shades of difference, obvious enough to the expert, between, say, a crystal from Bultfontein and one from Wesselson.

One or two opinions from authorities of greater or less repute may be cited :

" It is quite true that large parcels of diamonds from the various mines have distinctive characteristics, and it can be easily told from which mine a parcel of diamonds comes; but it is very difficult to tell in which mine a single stone may have been found, though each mine has stones in a great measure peculiar to itself." (G. F. Williams, 'The Diamond Mines of South Africa,' p. 492, 1902.)

" The collective character of the stones found in each mine and in each part of a mine are distinctive, but single stones of every quality occur in all mines. Thus, though it may be impossible to state the particular mine in which a single stone was found, yet an experienced Kimberley diamond merchant would have no difficulty in naming the mine, or portion of a mine, from which a parcel of stones had come, provided that the parcel formed a fair sample of the yield of that particular deposit." (Bauer, 'Precious Stones,' English edition by Spencer, p. 212, 1904.)

" Most pipes were formed by a number of successive eruptions which gave rise to more or less well-defined columns of blue ground. Within any particular column of this nature the diamond content of the pipe rock is generally found to be remarkably uniform, but adjacent columns may differ markedly not only in their yield, but by producing diamonds of a distinctive character. So pronounced in some instances are the differences between the diamonds yielded by different columns

of blue ground that experts can with[out] difficulty determine not only from which mine, but from what portion of the mine, a particular parcel of stones has been derived." (P. Wagner, 'The Diamond Mines of Southern Africa,' p. 149, 1914.)

We propose first of all to amplify these three statements, and to illustrate them as far as it is possible to do so. Further references to the volumes in which they are contained will give the pages only, the titles will be understood.

Below are given representative assortments, in percentages, of parcels of diamonds from the Bultfontein, Wesselton, and Dutoitspan mines. Representative assortments for De Beers Mine, and for Kimberley Mine, separately, cannot well be given, since in the days when these two mines were in full swing the produce of the two was mixed together before being sorted. A specimen Pool assortment (as it is called) of De Beers and Kimberley diamonds mixed together is, however, given.

1. *Representative Assortment of Bultfontein and Wesselton Diamonds, in Percentages.*

	Bultfontein.	Wesselton.
Size of Parcel	70,208 cts.	59,600 cts.
A. <i>Close Goods.</i>	Per cent.	Per cent.
Blue Whites	·02	·30
Fine Whites	2·06	2·64
Whites	·55	1·16
First Capes	·31	1·01
Second Capes	·06	·30
Byes	·03	·03
Yellows	·03	—
First Fancies	1·20	—
Second Fancies	·60	—
B. <i>Irregulars.</i>		
Whites	2·85	3·21
Capes	·24	·71
Byes	·09	·02
Yellows	·01	—
C. <i>Spotted Stones.</i>		
Blue Whites	·02	·35
Whites	2·84	1·92
Capes	·68	·66
Byes	·11	—
Goods	—	·85
Good Capes	2·28	·19

	Bultfontein. Per cent.	Wesselton. Per cent.
Second Capes	2.74	.62
Darks	1.71	.57
Dark Capes24	—
Dark Byes04	—
Blacks	1.52	—
D. <i>Brown Stones.</i>		
Light Browns	—	1.74
Fancies	—	.75
Coated Stones	—	.20
E. <i>Flats.</i>		
Whites	1.27	2.75
Capes14	.43
Byes10	.05
Greys	3.43	4.72
Darks	1.05	1.13
F. <i>Cleavages.</i>		
Blue Whites14	.04
Fine Whites	2.67	3.06
Fine Capes19	.71
Byes03	—
Yellows01	—
Goods	2.15	.74
Good Capes25	.32
Coloured70	.74
Greys	2.70	1.61
Darks	4.31	1.17
Blacks	4.74	—
Black Rejections	11.09	4.43
Light Browns22	.42
Browns	3.15	9.46
G. <i>Rejection Chips.</i>	—	3.65
H. <i>Rubbish.</i>	23.55	27.03
I. <i>Bort.</i>	18.00	20.31

2. Representative Assortment of Pool and Dutoitspan Diamonds, in Percentages.

	Pool. Per cent.	Dutoitspan. Per cent.
Size of parcel	30,760 cts.	30,900 cts.
AA. <i>Close Goods.</i>		
Crystals07	.44
Capes07	.26

	Pool. Per cent.	Dutoitspan. Per cent.
Byes	·13	·41
Light Off-coloureds	·04	·23
Off-coloureds	·11	·94
Yellows	1·45	1·74
BB. <i>Spotted Stones.</i>		
Crystals)		{ ·54
Fancies)	3·20	{ 3·01
Byes	·27	·77
Off-coloureds	·56	·95
Yellows	1·98	1·79
CC. <i>Flats.</i>		
Whites)		{ ·07
Irregulars)	1·54	{ 2·90
Byes	·94	·95
Yellows	1·21	·68
Dark Yellows	·40	·07
DD. <i>Macles.</i>		
Ordinaries	·23	·03
Darks	·29	·12
Blacks	1·15	1·03
Rejections	1·23	1·14
Black Rejections	1·11	·46
EE. <i>Rejection Stones.</i>		
Ordinaries	·13	·83
Light Yellows	1·07	2·33
Yellows	·86	—
Darks	·76	2·23
Blacks	·44	1·04
FF. <i>Cleavages.</i>		
Whites	·62	1·05
Capes	·75	1·33
Byes	2·19	1·23
Yellows	1·84	1·93
Ordinaries	1·07	·77
Darks	·42	·77
Blacks	5·46	7·78
Rejections	·67	2·15
Black Rejections	10·97	8·83
Light Yellows	5·78	8·00
Yellows	3·42	—
Light Browns	·70	·48

	Pool. Per cent.	Dutoitspan. Per cent.
Browns	28	39
Dark Browns	4.81	7.00
GG. <i>Rejection Chips</i>	13.04	7.53
HH. <i>Rubbish</i>	19.80	15.45
II. <i>Bort</i>	8.93	10.33

Note.—Some caution must be exercised in making a first approximation to an interpretation of these percentages. It would not be fair, for example, to translate them at once into carats, and thence to argue that Wesselton produces more good white stones than Bultfontein does just because the numbers opposite "whites" are greater as a rule for the former mine. Actually the percentages of whites in the upper classes of Bultfontein are diminished because of the large number of Bultfontein white, or faintly tinted, diamonds which contain black spots—as may be seen in Class C, and especially in Class F, sub-classes greys to black rejections inclusive. Were these spotted stones (which are not numerous represented in the Wesselton yield) not in existence, Bultfontein would show the higher percentages of good whites.

Explanatory Notes on the Assortments.

A. *Close Goods* are diamonds of good colour and symmetry and free from all blemishes of spots or flaws. In this, as in all the other classes, the sub-classes stand in the order of merit; thus a white is esteemed more than a cape, a bye more than a yellow. The sub-classes are further sub-divided into lots arranged according to size.

B. *Irregulars* resemble close goods in all respects excepting that their symmetry is inferior.

C. *Spotted Stones* resemble close goods excepting that they contain internal white or black spots.

D. *Brown Stones* are diamonds of good symmetry and free from blemishes, excepting that they have a greater or less brown or smoky hue. In the case of the light brown stones the colour is pretty equally distributed throughout the mass, as it also is in the case of the fancies (see also under A, Bultfontein) which are mainly brown. On the other hand, the smoke-brown tint of the coated stones is often confined to the surfaces, and is due to a thin deposit of opaque material, probably caused by corrosion. Coated stones are what is called "speculative" stones, since they may be good or bad inside. Pretty often they are very good indeed. They always command a good price. Bultfontein fancies differ from Wesselton fancies.

E. *Flats* differ from close goods in the matter of symmetry, namely, that their thickness is small in comparison with their length and breadth.

Some of these are no thicker than a visiting card. Macles (for the definition of which see any standard work on crystallography) go into this class. Mineralogists and crystallographers, outside the diamond trade, commonly give the name "spinel twins" to macles. The term is not without objection, because it is calculated to make the multitude think that a macled diamond is really a spinel. It is better to confine the name "macle," in accordance with Kimberley practice, to stones in which the crystallisation has taken place right- and left-handed from a common basal plane without interpenetration from either, and to use "twin" for interpenetrating crystals.

F. *Cleavages* will be dealt with at some length later on. Here it is enough to say that the first five sub-classes—from blue whites to yellows, that is—are of good quality, colour, and shape; whereas the remaining sub-classes, from goods to black rejections, are inferior in all ways, and in the majority of individuals are spotted. Brown cleavages bear much the same relation to brown stones that the first five sub-classes of cleavage bear to close goods.

G. and H. *Rejection Chips and Rubbish* are made up entirely of diamonds so defective in the requirements of a passable stone that no place can be found for them in any of the higher classes lest they spoil the look of the whole parcel. They comprise diamonds of all sorts other than true bort—the ragtag and bobtail of each. A good quantity of this stuff goes into cheap jewellery; much of it is used for industrial purposes, *i. e.* for graving tools, glaziers' diamonds, and watch pivots. Bultfontein does not produce so many small diamonds, averaging, say, a dozen to the carat, as Wesselton does, which is one good reason why it has not a class of rejection chips of its own. Such rejection chips as it has go into the rubbish.

I. *Bort* is mostly converted into powder for grinding and polishing purposes. There is a large demand for the best pieces for use in rock drills. Notwithstanding that bort only averages 5s. or 6s. a carat, really good "shot bort" will fetch as much as £4 or £5. Some of the distinctive types of bort will be described presently. Diamonds not good enough even for the rubbish go into this class.

AA. *Close Goods*. See the remarks under A.

BB. *Spotted Stones*. See C. In sorting Pool or Dutoitspan goods for the market, fancies find their best home in this class.

CC. *Flats*, here, really contain a few macles; but most Pool and Dutoitspan macles do not harmonise as well with the flats as Wesselton and Bultfontein macles do, and so we have a separate class of—

DD. *Macles* which are characteristic, in colour and shape, of the former mines. They tend to a greenish-grey, are often much spotted, and have, with few exceptions, rounded edges sloping outwards (not inwards like spinel twins) to the common basal plane.

EE. *Rejection Stones* are inferior spotted stones not good enough for BB.

FF. *Cleavages.* See F.

GG, HH, II. *Rejection Chips, Rubbish, and Bort.* The remarks under G, H, I, apply in general in these cases also.

It is to be noted that these assortments are on a strictly commercial basis. They have the market in view, and the market only, therefore they cannot fully answer all the questions that science would put; but they give useful testimony as to the outstanding points of difference between the parcels of diamonds from various sources. Roughly speaking, one system of sorting does for both Wesselton and Bultfontein, and another system for both Pool and Dutoitspan. All the same, that is not to say that for any assigned class, common to the two mines, the diamonds belonging to that class from one mine are the counterparts of those from the other. The Bultfontein cape stone, *e.g.* is not quite the same as the Wesselton cape. A Dutoitspan yellow has a somewhat different glow from a De Beers yellow. Bultfontein browns are sorted in with the fancies. Both they and the Wesselton browns are less full in colour than Pool browns; indeed they are smoky rather than brown. Still more diverse are Wesselton whites and Dutoitspan whites. Again, the Bultfontein yield sorts into more classes than does the Wesselton yield, there being small classes of coloured diamonds in the former which are not represented at all in the latter, which is curious, seeing that a Bultfontein parcel has quite as white a tone as a Wesselton one. As it happens, many of the yellow diamonds, and particularly the larger ones, found in Bultfontein are what is called Dutoitspan yellows; that is, they have the Dutoitspan and not Bultfontein characteristics; and this fact, coupled with the occasional find of a diamond with Bultfontein characteristics at Dutoitspan, is thought to indicate some deep underground connection between the two mines (Hatch & Corstorphine, *Geology of South Africa*, second edition, p. 276, 1909; Wagner, p. 151). The superior quality of the Dutoitspan yield over that of the Pool is well shown by the greater percentage of Dutoitspan diamonds in the upper classes. An excellent brief general account of the characteristics of the diamonds from the several mines will be found in Hatch & Corstorphine, p. 275, made up from notes supplied by A. Brink.

As to the denominations, it should be explained that "black" is not black in the sense in which lady novelists use the term when they rhapsodise over their heroine's priceless black diamonds. It simply means diamonds in which black spots are abundant. In Kimberley colloquial speech a black diamond is generally bort when it is not coal.

2. COMPARATIVE STATISTICS OF LARGE DIAMONDS.

One of the most noticeable points of difference between the diamonds produced from this or that mine in the Kimberley group is in the matter

of size. Taking all diamonds of 10 carats each, or greater, we have the following comparison:

Year.	Mine.	Total production in thousands of carats.	Average size of diamonds of, or exceeding, 10 carats.	Ratio of total weight of diamonds exceeding 10 carats to total production.	Number of diamonds of, or exceeding, 10 carats each in every 100,000 carats produced.
			Carats.	Per cent.	
1898	De Beers	1897	18.2	12.25	674
1898	Kimberley	652	17.0	11.65	684
1912	Wesselton	580	15.5	2.32	140
1912	Bultfontein	893	14.9	.97	65
1912	Dutoitspan	504	20.0	17.02	850

Thus diamonds exceeding 10 carats each in weight are relatively infrequent at Bultfontein, and average small; whereas upwards of one-sixth of the Dutoitspan yield consists of stones averaging 20 carats each.

This is remarkable, seeing that these two mines are so close together that they could even be worked (though perhaps not very profitably) from the same main shaft, and possibly have, as we have just said, some underground connection. Comparing one mine with another, the rule is essentially: The greater the average size of diamonds exceeding 10 carats each, the greater the ratio of their total weight to the total production, and the greater their number in every 100,000 carats produced. Kimberley Mine deviates slightly from the rule in the second particular.

3. LESS OBVIOUS DIFFERENCES.

Besides all this there are the subtle differences, previously alluded to, between the diamonds from different mines, which, though they (the diamonds) may be called by the same name, have yet an undoubted dissimilarity one from the other made up of almost indefinable distinctions of lustre, brilliancy, crystallisation, appearance, texture, and general tone. Among other things there is the characteristic rippled surface of a Dutoitspan diamond as compared with the smooth surface, often with numerous triangular indentations, of a Wesselton diamond, and as compared with another from Bultfontein. The passages cited at the beginning—which are not really independent statements—really understate the case. A person not in daily contact with Kimberley diamonds would naturally not be able to recognise these differences, but an expert can do so more readily than might be supposed. Single stones of every quality, it may be added, do *not* occur in all mines.

Slight differences have been suspected, and may indeed exist, between the lustre of diamonds won from different depths in the same mine. The evidence, however, is hardly stronger than that of mental impression based

upon memory. While the expert may honestly think that the diamonds now mined are brighter, or less bright, than they used to be, there is just the possibility that his eyes may have deteriorated or his memory been at fault. Moreover, it is not practicable in the process of mining to keep the diamonds belonging to different columns of the same pipe entirely separate, and also the columns may not be of the same relative cross-section throughout, so that at one level in the mine the output may consist of a greater proportion of diamonds from one column than it does at another level.

4. OCTAHEDRA.

The perfect octahedron is a rarity. Perhaps a few dozen have been found in the mines of the Kimberley group in forty years, and one at any rate of these was found entirely enclosed in another diamond—which suggests the possibility that any octahedron of a perfect shape, with sharp edges that is, may have spent its existence, until it reached the surface, inside another diamond. Bauer's statement that "the edges and corners of the crystals [of Kimberley diamonds] are always perfectly sharp, not even the faintest trace of rounding can be detected," is not even approximately true. Yellow octahedra are rare. The common bright yellow diamond is nearly always rounded.

5. COLOURS.

Orange-coloured diamonds are only found, as a rule, in the Wesselton Mine. This is curious, too, for *a priori* one would have expected to find them in mines where yellow diamonds abound, *i. e.* in De Beers, Kimberley or Dutoitspan. They are practically always in misshapen fragments, like their associated minerals garnet and zircon, and doubtless owe their condition to much the same causes as wore or broke down those. They are nearly always small, not often exceeding 1 carat, and not more than half a dozen or so of them are found in a month. They keep their original colour pretty well, as a rule, after cutting. In the collection of brilliants exhibited in the Kimberley office of the De Beers Company is a beautiful rich orange-coloured diamond weighing 2 carats, cut from one of the largest of such fragments yet found. G. H. Smith's assertion ('Gem Stones,' p. 151, 1912), that the "Wesselton Mine yields a large proportion of flawless octahedra, but above all a large number of beautiful deep-orange diamonds" is perhaps approximately true as to the first half, but not reasonably true as to the second.

Pale lilac fragments are met with at rare intervals, and one reddish lilac diamond has been found and preserved, which, in shape, is like a long, flat bean, and yet has natural, though possibly secondary, faces.

Now and then stones with a greenish cast are found, chiefly in De Beers and Kimberley. They range in colour from a faint chrysolite green to sage.

The lighter greens are nearly always found among rounded octahedra, and the colour appears to be concentrated in the crystal corners, and hence almost entirely disappears in the cutting, as we are told and should half expect. The sage greens are disappointing as brilliants; though they are uncommon they are not prized, for they have no life and little beauty. Steel blue and sapphire blue diamonds, like those from the Premier Mine in the Transvaal, have not been found near Kimberley.

Brown diamonds ranging in colour from smoke-brown to chocolate are found, those of the latter colour rare and precious as brilliants when they possess "fire." Beautiful pale brown and autumn brown stones, of large size up to a hundred carats but of flat and irregular habit, used to be found in the Pool. Inexperienced diggers have been known to throw them away, mistaking them for "Dutch bort" (zircon). One fine piece, worth upwards of £200, was ascertained to have been thrown away by one *débris* washer after another before it finally reached the market. Such stones cut very well. Their early history in the pipes must have differed much from that of the majority of diamonds.

After all, if the truth must be told, the common yellow diamond is by far the most beautiful of all, and it would be esteemed the most if it were less common. And it has this advantage, which a good many white stones have not—that it is full of fire by day or by night. The great Tiffany lemon yellow brilliant is a proof of this affirmation. Judging by descriptions and by glass copies, it is a Dutoitspan diamond.*

Stones showing slight colour effects under the polariscope are not at all uncommon, but they are conceivably not so common as is sometimes asserted. The statement that nearly all Kimberley diamonds show signs of great internal strain is no more than assumption based on casual examination of a comparatively few specimens, and is scarcely more likely to be generally true than the old idea, still believed by many—that all diamonds phosphoresce after exposure to sunlight. How many South African diamonds ever go near a polariscope?† (But see Sir William Crookes's Kimberley lecture on Diamonds, 1905.)

6. BORT.

Perhaps the bort class is the most interesting of all. "Bort" as a commercial term includes inferior representatives of all the other classes besides true bort. True bort consists of crystalline aggregates of tiny particles

* The Tiffany diamond is commonly said to have been found in the Kimberley mine in 1878. The diamond fields newspapers of that year, however, do not report such a find.

† Crystals and crystallisation have always provoked random theories. Note, e.g. the old and utterly unobservant idea, endorsed by no less a philosophic autocrat than the 'Encyclopædia Britannica' that "hoar frost is nothing but dew turned into ice by the coldness of the air" (3rd edition, 1797).

arranged anyhow in rounded lumps. The prevailing colour is grey, and in the mass it is opaque. But very elegant cubes and spheres of this material, ranging in colour from fawn to dove, occur which are quite translucent. Typical Wesselson bort is a sort of semi-translucent substance, black by reflected light, with a lustre remotely reminiscent of tourmaline. We meet with occasional fragments of black bort, an amorphous material perhaps more nearly allied to carbonado than to true diamond. This occurs in greater abundance in the Transvaal workings than at Kimberley, and pieces have been found on the Vaal River and at Jagersfontein. It occurs in the form of irregular lumps, and is not favourably regarded as diamond at all in the market. A beautiful and unique specimen of this class, in shape generally rounded but with the corners sticking out, much like a De Beers' yellow stone in habit, is in my keeping. D. P. McDonald has described a piece of this kind of bort in "Notes on a Form of Black Diamond from the Premier Mine" ('Trans. Geol. Soc. S. A.,' 1913). This material is now known as framesite.

Bauer's statement that 90 per cent. of South African bort comes from the Kimberley Mine was wrong when he wrote it, and is still more wide of the mark now.

7. STEWARTITE.

The most interesting and most important individual of all the very diverse bort family is the substance for which the name "stewartite" has been adopted, and which claims a section for itself. This was discovered by J. Stewart. It is of a steel-grey colour, with a dull steely sheen, and is more fibrous in texture than the types of bort mentioned above. In hardness it equals the diamond, but its specific gravity is rather less.* It is a good conductor of electricity, like other borts. Its most remarkable property, though, is that it is magnetic and polar, and thus it has an important bearing on Crookes's theory that diamond has separated out from molten iron. Being magnetic, it must contain iron; and because it contains iron and carbon in association and is polar, we might almost call it a steel bort. Hitherto iron had only been found in infinitesimal quantities in the ash left after burning a diamond.

The properties of stewartite recall those of the fabulous, or alchemical, adamant of the Middle Ages, when a confusion of ideas (for it is difficult to conceive that there could have been an adequate knowledge of the facts) seems to have evolved an imaginary substance combining the properties of diamond and lodestone. Chaucer seems to have distinguished between diamonds and lodestones, though possibly not by intention, using the spelling "adamauntes" for the latter and "ademauntz" for the former. Thus (Morris's ed., 1886):

* S.G. = 3.45.

" Ryght as betwix adamauntes twoo
 of evene wyght, a pece of iren isette
 Ne hath no myght to meve to no fro."

('The Parlement of Briddes.')

And so in 'The Romaunt of the Rose.'

Whereas

"The dores were alle ademauntz eterne."

('The Knightes Tale.')

Gower uses "adamaunt" and "adamant" in our sense as "diamond." On the contrary, Shakspeare uses the same word ('M. N. D.,' II, 1) in the sense of lodestone, although his "hard-hearted adamant" has a flavour of the diamond about it.

The 'Oxford English Dictionary' has an interesting article on the word "adamant," and a number of quotations from English authors down to 1750, which prove how little was known at first hand, or by experiment, of the properties of either lodestone or diamond. By many the adamant was thought to be a sort of natural opponent of the lodestone, counteracting the attraction of the latter for iron. Others regarded the adamant and lodestone as identical, and thought that the diamond was the natural opponent of both.

There is a good specimen of stewartite in the McGregor Memorial Museum at Kimberley. The substance described by Wagner (p. 143) is, of course, not stewartite at all, but black bort, the same as, or allied to, the black bort mentioned in the previous section.

8. VARIATION OF QUALITY AND INCREASE IN THE OUTPUT OF CLEAVAGES, WITH DEPTH OF WORKING.

The question is often raised whether the average quality of the diamonds found in the different mines deteriorates with working depth. It is a difficult question to answer definitely, because the methods of winning the diamonds have improved with experience; so that small diamonds which might have been lost twenty years ago are now saved, and hence that the average value must decrease. In other words, the proportion of good stones to bad must decrease even if the larger ones are as good as ever they were. In the table following are given for each year 1902-16 the relative proportions for Bultfontein of—

- (1) *Stones*—i. e. Close Goods, Irregulars, Spotted Stones, and Flats;
- (2) *Cleavages*;
- (3) *Rubbish and Bort*;

the mean working depth from which the diamonds were won increasing in that time from about 300 feet to 850 feet. The first column gives the years, the second the number of carats in thousands (M) sorted for sale, while the others show percentages of each class.

Table Showing the Percentages of each Class of Diamonds from the
Bultfontein Mine.

Year.	Carats sorted.	Stones.	Cleavages.	Rubbish and bort.
	M.	Per cent.	Per cent.	Per cent.
1902-4	265	29.54	29.03	41.43
1905	291	30.84	34.87	34.29
1906	533	28.87	36.27	34.85
1907	506	26.61	35.73	37.66
1908	398	26.56	33.48	39.96
1909	670	25.05	32.01	42.94
1910	678	23.89	34.08	42.04
1911	778	23.87	35.70	40.43
1912	855	23.12	36.63	40.25
1913	787	22.25	36.30	41.45
1914-16	992	23.19	32.64	44.16

The first line is not perhaps fairly comparable with the others, since it represents the first years in which systematic work was begun by the De Beers Company, and no doubt some clearing up from previous irregular work by various small companies had to be done. But from 1905 onwards there seems to have been a pretty clear increase, on the whole, in the rubbish and bort of about 10 per cent. At the same time the cleavages have been very little affected, while the ratio of stones has decreased from say 30 to 23 per cent. If we assume that the increase of rubbish and bort is due to improved mining methods, and reduce the ratio of rubbish and bort accordingly, this would indicate that the quantity of stones found has not decreased much, if at all, and may indeed have increased somewhat.

With this there does seem to have been an increase in the actual yield of brown diamonds as compared with the rest. The following are the separate percentages of brown and other cleavages:

Year.	Brown cleavages.	Other cleavages.
	Per cent.	Per cent.
1902-4	3.92	25.11
1905	2.96	31.91
1906	3.45	32.80
1907	3.85	31.88
1908	3.91	29.57
1909	3.52	28.49
1910	3.84	30.22
1911	3.88	31.82
1912	4.15	32.48
1913	4.28	32.02
1914-16	4.38	28.26

Whether any of this result is due to psychological reasons is hard to say. Diamond coloration merges from one class to another not by steps but imperceptibly. And it might therefore be plausibly urged that some cleavages which formerly were not brown enough to be called brown are now so called; or that it is likely that improving markets have to some extent justified the promotion of individuals from the rubbish into the brown cleavages. Neither alternative has any trustworthy evidence in its favour, and my own impression is that the increase in the brown cleavages is real, the small deviations from a uniform increase showing no definite relationship to the deviations of rubbish and bort.

The next table gives similar information for Wesselson for the years from 1898 to 1916—namely, the relative proportions of—

(1) *Stones*—i. e. Close Goods, Irregulars, Spotted Stones, Browns, and Flats;

(2) *Cleavages*;

(3) *Rejection Chips, Rubbish, and Bort*;

the mean working depth from which the diamonds were won increasing in that time from 125 feet to 800 feet.

Table Showing the Percentages of each Class of Diamonds from the Wesselson Mine.

Year.	Carats sorted.	Stones.	Cleavages.	Rejection chips, rubbish and bort.
	M.	Per cent.	Per cent.	Per cent.
1898	378	31·72	17·06	51·22
1899	447	29·53	15·47	55·00
1900	308	28·14	15·51	56·35
1901	484	28·08	16·33	55·59
1902	541	26·65	17·76	55·59
1903	380	25·97	17·86	56·18
1904	612	23·86	19·20	56·93
1905	567	24·98	19·39	55·63
1906	590	24·41	20·56	55·03
1907	472	26·81	23·67	49·52
1908	397	25·95	23·86	50·19
1909	618	25·56	24·72	49·72
1910	525	25·81	25·61	48·57
1911	409	24·39	23·43	52·18
1912	522	24·17	24·87	50·96
1913	602	23·05	25·20	51·75
1914-16	634	23·38	23·05	53·57

Here we find, so far as the statistics can show it, a decrease in the percentage of stones of about 7 per cent. (about the same as that of Bultfontein in a little shorter time); an increase of cleavage of about the same amount; and a remarkable fluctuation in the percentages of rejection chips, rubbish, and bort, the curve of these showing, on the whole, a maximum in 1904, followed by a minimum in 1910, and after that a rise to the present time.

Curiously enough, the brown stones show a uniform decrease with time, whereas the brown cleavages show a definite increase, thus:

Year.	Brown stones. Per cent.	Brown cleavages. Per cent.
1898	2.82	5.17
1899	3.10	6.35
1900	2.97	6.44
1901	3.55	6.20
1902	3.33	7.15
1903	3.14	7.22
1904	2.62	8.19
1905	2.49	7.63
1906	2.37	8.21
1907	2.65	9.65
1908	2.29	9.61
1909	2.28	10.70
1910	2.17	10.84
1911	1.97	9.64
1912	1.89	10.13
1913	1.90	10.57
1914-16	2.12	10.13

This is an extraordinary result, and requires more material for its full elucidation than has yet been accumulated. It would appear from the face value of the relative numbers that brown stones have shown a greater disposition to break up in the deeper levels of the Wesselton Mine than in the upper ones. Also that brown diamonds (stones and cleavages together) have increased by about 3 per cent. in eighteen years, the fluctuations showing some, if small, agreement with those of rejection chips, rubbish, and bort.

Comparing stones other than brown with cleavages other than brown, we have—

Year.	Stones not brown. Per cent.	Cleavages not brown. Per cent.
1898	28.89	11.89
1899	26.43	9.12
1900	25.17	9.07

Year.	Stones not brown.	Cleavages not brown.
	Per cent.	Per cent.
1901	24.53	10.13
1902	23.32	10.61
1903	22.83	10.64
1904	21.24	11.01
1905	22.49	11.76
1906	22.04	12.35
1907	24.16	14.02
1908	23.66	14.25
1909	23.28	14.02
1910	23.64	14.77
1911	22.42	13.79
1912	22.28	14.74
1913	21.15	14.63
1914-16	21.26	12.92

Here we see that there has, apparently, also been a disposition for these whiter diamonds to break up more freely at the greater depths, although not to the same extent as in the case of the brown diamonds. For whereas in the five years 1899-1903 67 per cent. of all brown diamonds was brown cleavage, in the five years 1909-13 the cleavages had increased to 84 per cent., an increase of 17 per cent., the corresponding percentages for diamonds other than brown being 29 and 39, an increase of 10 per cent. Moreover, in absolute numbers, the chances preponderate that a brown diamond will be a cleavage, whereas the odds are quite the other way when the diamond is not brown. There is pretty well as much brown cleavage as there is cleavage not brown; the stones other than brown outnumber the brown stones by ten to one. Bauer, in one of his characteristic inaccuracies, has stated that in the case of Cape diamonds "it is a remarkable fact that these cleavage fragments are nearly always white—that is, colourless, or, at least, very faintly coloured; fragments of a dark colour, or of a decided yellow, are extremely rare, so that we must conclude that such stones offered greater resistance to fracture than did the colourless diamonds" (p. 209). The above statistics show just the opposite—that it is the colourless diamonds rather which resist fracture, as compared with brown, at any rate, although, as we shall see when we come to speak of Pool and Dutoitspan diamonds, yellow diamonds are perhaps more resistant than white. Apart from any question as to the influence of colour upon the cohesion of a diamond, we should naturally expect cleavages to appear somewhat lighter in colour than stones, for the same reason that many cut diamonds appear lighter in tint than they did when in the rough. A good deal of the colour in some rough stones seems to be concentrated in the crystal corners from

which it is reflected inwards, so that if the stones were to be broken the fragments would assume a lighter tint. We have also to remember such a fact as that the deepest coloured glass appears lighter in colour when broken into small enough fragments, and quite white when reduced to powder. Diamonds may look lighter in colour because they are broken; they do not break because they are lighter.

9. CLEAVAGES.

One point to be noted concerning what are called cleavages is that some of them are not broken diamonds at all. Wodiska's statement ('A Book of Precious Stones,' 1910) that "cleavage" means broken stones is too unpromising. "Cleavage" is a trade term which indicates not only diamonds now broken, but those which will have to be broken before being finally cut into gems—potential cleavage, in fact. All very misshapen stones and interpenetrating crystals, besides broken pieces, are cleavages in the eyes of a Kimberley diamond merchant, just as all flesh is grass, and so are feathers too, in the eyes of the philosopher. Bultfontein produces numbers of these interpenetrating crystals, or cross-grained stones, as they are called. Some of them take strange forms: a fine white one in my collection is like two mace-heads, with sharp, projecting points, joined together. The term "cleavage" does not apply, however (as Bauer says it does), to crystals containing spots which would have to be cleaved before being cut. A crystal containing spots is a "spotted stone," and is denominated dark, black, or black rejection, according to its quality. Some of these so-called cleavages have evidently crystallised in a tight corner, judging by their distorted outlines and by the fact that sporadic specimens appear to have surfaces of attachment like quartz. Others appear to have been broken at an early stage, and to have undergone further growth or suffered resorption. It is truly remarkable how many of these so-called cleavages seem to have been naturally refaced either with a nascent or with a pocky surface. The triangular indentations indicating growth—so common on the faces of Wesselton octahedra—do not, nevertheless, seem to occur on these naturally renovated cleavage faces; instead, striations like those on yellow diamonds are the rule.

10. BROKEN DIAMONDS.

The origin of the numerous broken fragments of diamond is somewhat of a mystery. G. F. Williams (p. 492) remarks that—"Some observers claim that the broken diamonds which are extracted are broken during the process of winning them. It is admitted that diamonds may be broken in the process of mining and the subsequent operations of winning, but these cases are exceptional. Fragments of diamonds are very frequently found embedded in the blue ground, and there is no doubt in the mind of anyone

who has had practical experience in finding these fragments that they were not crystallised where they are found." On the substructure of this evidence Wagner (p. 164) and Bauer (p. 195) assign the origin of Kimberley diamonds to a deep-seated plutonic rock, claiming that the broken fragments owe their condition to violent eruptive outbursts which shattered country rock and blue ground alike.* The latter, however, stultifies his argument to a certain extent in his very next paragraph, where he states that the minerals and rock fragments in the blue ground show no signs of wear and tear. It is hard to see how the diamonds could have suffered so much damage and their kimberlite surroundings none at all. As a matter of fact, better authorities tell us that the rock fragments in the pipes do show a great deal of wear and tear. (See A. L. du Toit, "Geological Survey of the Eastern Portion of Griqualand West," in the Eleventh Annual Report of the Cape Geological Commission, 1906.) Du Toit, it may be noted, provisionally accepts the theory that the shattering of the kimberlite accounts for the fracturing of diamonds, but observed that garnets have not suffered to the same extent (p. 151).

11. SPONTANEOUS BREAKING.

A more attractive theory, and one repeatedly quoted from one to the other by writers on the diamond, is that certain classes of diamonds frequently tend to break of themselves. G. F. Williams (p. 500) is responsible for the opinion that "light brown, smoky diamonds often crack on exposure to the dry air, but they will remain intact if kept in a moist place. The cracking is, therefore, more probably the result of heat or drying than of tension or inward [[?] interior] pressure. It is possible, however, that the great heat to which the diamond is exposed when brought to the surface may expand contained gases sufficiently to crack the stone." Crookes, on the other hand (Kimberley Lecture), seems to attribute the fractures to sudden lowering of pressure in the space surrounding the diamonds, and speaks of consequent explosions. G. H. Smith (p. 131), following Crookes, says that "so great is the strain that many a fine diamond has burst to fragments on being removed from the ground in which it has lain."

Unfortunately, the evidence for this particular theory, as a complete theory, is not strong nor very extensive. What, at first sight, might look like a consensus of scientific opinion is, in reality, gossip handed on from one to the other by nearly all, and is by no means the outcome of independent research. Personally, I have met plenty of people who have *heard* of the bursting of smoky diamonds without ever having witnessed such a disaster with their own eyes, and the story of the custom of sending them to England inside potatoes is almost a chestnut. Certainly the De Beers Company never pack any of their millions of diamonds in potatoes. A

* Cf. also L. J. Spencer in 'The World's Minerals,' p. 46, 1911.

possible case of spontaneous breakage is known in the De Beers' sorting-office, and another, in which a crack was found in a diamond where no crack was observed—and the chances are, did not exist—before. Besides this, there are many brown and other fragments which do possess very fresh-looking cleavage faces, and so might have done as the frog did in the fable.

And then there is the deduction from the statistics, previously remarked upon, that the proportion of brown cleavages to brown stones is unduly high. All this hints at the possibility of some measure of early or late spontaneous disintegration, but no more. Moreover, a great many broken diamonds are not brown or smoky at all. Anyway, diamonds do not *explode* of themselves under meteorological conditions. It is said that certain philosophers, acting under the orders of the Grand Duke of Tuscany, once caused a diamond to explode by exposing it to the sun's rays in the focus of a powerful lens two-thirds of a Florentine ell in diameter; but in this case, since the diamond was of a good size—nearly ten carats in weight—the result might have been caused by unequal heating rather than by the terrific heat alone.

If heat will not cause a diamond to shiver (!) according to Crookes, and moisture will prevent such a catastrophe according to Williams,* it is not easy to understand what any "cunning dealer," wishing to buy but taking no unnecessary risks, has to gain by encouraging sellers to handle their goods freely or carry them in their warm pockets, which is what we are told the cunning dealers do; and cunning indeed they must be if they can so contrive, and foolish the seller who could be so circumvented. It is to be suspected that much of the myth that has collected round the nucleus of an occasional accident to a smoky diamond is derived from the old story of Albertus Magnus, that a diamond immersed in the fresh, warm blood of a goat (especially if the animal had previously browsed on parsley or drunk wine) would burst. Pliny, it will be remembered, had a similar idea—namely, that the blood of a billy-goat had power to lessen the molecular cohesion of the adamas stone, which stone seems to have been diamond but may have been something else: "The blood, however, must be no otherwise than fresh and warm; the stone, too, must be well steeped in it, and then subjected to repeated blows; and even then it is apt to break both anvils and hammers of iron if they be not of the finest temper" (xxxvii, 15).

With reference to the hypothesis of heat causing a diamond to break or explode when it is brought to the surface, the highest temperature to which it would be subjected above ground would be experienced on the depositing floors under the sun's rays, say about 60° C. (140° F.). The temperatures underground at the greatest depths yet reached are not so high as this,

* A few experiments have been made with the object of ascertaining whether a moderate heating (up to a temperature of 100° C. in water) will create strain in a diamond or vary a strain already there; but with no proof as yet that it can.

probably not touching 40° C. A large flow of water met with in the main rock tunnel at a depth of 1525 ft. in the Kimberley Mine had a temperature of about 30° C. (84° F.).

The theory that diamonds were broken by violent movements of the kimberlite does not explain the cause of the breakages among diamonds found in gravels, unless we may conclude that these diamonds were brought to the surface in outbursts of plutonic energy. Perhaps they were not so brought.

Again, a yellow diamond recently examined, having a very striated surface which prevented its interior from being properly seen, showed a crack running half the way round the outside of the stone. The polariscope indicated a great state of strain somewhere near the centre of the stone at a place where the crack seemed to extend inwards to. Now, if the expansion of contained gas had caused the stone to split, the gas must have escaped through the crack. Why, then, the continued strain? Again, where could the internal strain have come from if the stone had been broken by concussion in machinery or pipe? The De Beers Company has a diamond with a percussion figure like a clock-face under its surface, and in this case the shock which made the figure has set up no internal strain. We shall see in the following section how the strain and crack in the above yellow diamond have probably arisen.

12. SUGGESTED REASONS FOR BREAKING; MINERAL INCLUSIONS, ETC.

The suggestion now to be put forward, with all deference to better authorities, is that the circumstance of broken diamonds on a large scale is not alone due to:

- (1) Either the spontaneous breaking up of those that are smoky or brown, or any other colour;
- (2) Or to breaking in the process of mining and winning;
- (3) Or to violent movements of the kimberlite in past time;
- (4) Or to the expansion of contained gases within the diamonds whether by heating or relief of pressure outside,

though each or any of these actions may have been of some effect; but principally to the energy exerted by the mineral inclusions so often contained in diamonds. These inclusions are most frequently garnet, and there are, besides, zircon, ilmenite, iron pyrites, and (possibly) chrysolite.* There are also inclusions of what may loosely be called *splotches* of graphite (very numerous), oxide of iron, and chrome diopside (rare). Some of the black inclusions may be either ilmenite or hæmatite, or graphite, but they are

* Or enstatite: "A bright green variety occurs in large cleavable fragments in the diamond-bearing detritus of Colesberg Koppje in the Transvaal"—H. Bauerman, *Text book of Descriptive Mineralogy*, 1897. Some of this material is harder than window glass.

so small that specific gravity tests (the only tests that are available without breaking the diamond) are not delicate enough to determine which. Diamonds possibly tinted by diopside have turned up now and then; but only one hitherto in our mines, containing a definite splotch of the material. But some years ago a beautiful specimen of a diamond was found in the Voerspoed Mine (O.F.S.), which contained a number of bright green splotches, doubtless diopside. This stone is now in the possession of G. F. Williams, and is probably peerless of its kind for beauty. A genuine crystal of chrome diopside has yet to be found in a Cape diamond; although it has been met with in bort.

Bauer (p. 192) states that "until recently no diamond had ever been observed attached to another mineral in such a way as to suggest that the two grew side by side at the same time. The discovery, however, of a diamond crystal attached in this way to a garnet shows that such a growth does take place, though rarely." This assertion besides being bad science is bad history. The phenomenon is neither rare nor new. Evelyn relates in his diary how on Michaelmas Day, September 29, 1645, he visited the collection of the noble Venetian Signor Rugini where he saw "above all a diamond which had a very fair ruby (? garnet) growing in it."

In the majority of cases the garnet, or other inclusion, is not actually inside the diamond, but is set more or less deeply in a cleavage face. Also a very great number of cleavage fragments contain somewhere in their fractured surfaces small rounded holes or indentations such as could scarcely arise in breaking unless the holes or indentations had been there first, with something inside them. This latter circumstance indicates that diamonds containing inclusions have been more frequent in nature than we should infer from even the considerable number found with the inclusion present. Some of these holes may possibly have contained gas or liquid.

In trying to interpret the facts we have to bear in mind that the thermal expansion of pretty well all crystals, saving those of the beryl family, at ordinary temperatures, is much greater than that of the diamond. Between 0° and 100° C., for example, the coefficients of cubical thermal expansion are for:

Zircon	0.2835
Garnet	0.2543
Quartz	0.3530
Diamond	0.0354

Whether these relative values would hold under plutonic conditions of heat and pressure is perhaps uncertain, though the coefficient of thermal expansion for diamond is said by Miers to increase rapidly at high temperatures and to diminish rapidly at low ones. For our present purposes very high temperatures need not concern us; nothing higher in fact than the com-

paratively low temperature at which garnets fuse, since there does not appear to be any evidence that the garnets of the Kimberley Mines have ever attained this temperature. "One character," says Church ('*Precious Stones*,' p. 64), "common to all garnets save Uwarowite is their fusibility before the blowpipe; they thus yield a vitreous mass which is of much lower density than the original garnet before fusion." This refers to experiments in air, of course, and things may be somewhat different under pressure deep down in the earth. In any event, the garnets of our mines have probably not been subjected to very high temperatures because none seem to have been found of the requisite low specific gravity. Kimberley garnets vary in specific gravity, it is true, but none tested in the De Beers office have been anything like so low as 3, the value for vitreous garnet. We shall, doubtless, be on the safe side in assuming that the thermal expansion of garnet, zircon, etc., under temperature conditions that are reasonably probable, is always greater than that of diamond, though not necessarily greater by a constant multiple.

Two alternatives are conceivable. First suppose a diamond to have crystallised about a foreign body, such as a garnet, and that at some subsequent time there was an increase of temperature in the surrounding magma. It is clear that a greater relative expansion of the inclusion would put the diamond into such a state of strain that, if the magma were not solid, the diamond might give way and break. That there has been such an elevation of temperature is rendered not improbable by the large number of diamonds showing signs of corrosion (? Luzi's figures), and by the coatings of graphite on others; possibly also by the phenomena of flaked diamonds. In the De Beers collection is one of these, a stone the greater part of which is in thin opaque grey laminae, like an oyster-shell.* The state of this diamond was for a long time erroneously attributed to shattering by dynamite, but it is more likely to have been caused by overheating in the pipe during some past time. Many Premier diamonds, also, show incipient local flaking ("feathers") on their surfaces. Some of the diamonds belonging to the Emperor Francis I are said to have split into thin flakes when heated in a crucible. Elevation of temperature may also be indicated by the opaque white material, unaffected by hydrofluoric acid, occasionally found in local patches on diamond surfaces. Quite a large area of a certain lump of black bort from the Premier Mine was thus covered. A similar-looking stuff—perhaps the same—occurs, however, in some of the cavities in the cleavage faces of diamonds and round enclosed garnets, which makes it difficult to understand how it could be produced by heating; for the heat required should presumably have destroyed the diamond surface outside before it could reach the cavity.

* The crystalline part of this diamond is a non-conductor of electricity, but the laminated part conducts as well as bort.

Another guess that might be made is that the diamond crystallised about the garnet at some pretty high temperature, and that the relative expansion of garnet as compared with diamond was less than it might have been at some later lower temperature. In this case it would be the cooling as plutonic activity died away that would set up the fracturing.

Whether we accept either alternative or neither, it is certain that diamonds do cleave freely across foreign crystal inclusions. Here is a case in point: A very fine diamond, weighing 42 carats and valued at over £600, was recently found in the Wesselton wash. It is a fair-shaped octahedron, as Cape octahedra go, and is much indented with the shallow triangular depressions characteristic of its mine. It contains the most lovely inclusions one can imagine—one apparently a small garnet, another a honey-coloured zircon, just deep enough in colour to show, *very* faintly, two colours in the dichroscope, of fair size. Through, and surrounding the position of, the supposed garnet is an extensive crack, while both above and below the zircon, and tangential to it, are similar cracks. These cracks are parallel to contiguous faces of the diamond, and neither reaches the surface of the latter.* The zircon with the two parallel cracks touching it exhibits an odd resemblance to an airplane in flight. Under the polariscope a state of great internal strain is manifested over a large area in the region of the zircon by a most magnificent coloration, while the zircon itself is equally splendid. Words indeed are insufficient to describe the chromatic beauty of the polariscopic view. Curiously, there appears to be no strain to speak of in the vicinity of the crack through the garnet, as though the strain which once must have existed there and caused the crack had been relieved.

Of course, one would expect more strain in general about a zircon inclusion than about a garnet one because of the somewhat greater coefficient of thermal expansion of the latter, especially along one of its crystal axes. This outstanding specimen illustrates very well the force of the two alternatives mooted above. For as internal strain still persists in spite of the present moderate temperature of the stone, it would appear that either the diamond must have crystallised at a surprisingly low temperature or at some high one at which its coefficient of thermal expansion must have been relatively greater, compared with that of the zircon, than it is now.

Arising out of the condition of the specimen in question an interesting point emerges:

Some broken diamonds, as we have said, show rounded holes in their cleavage faces, but others do not. Now we cannot conclude that those which do not show the holes were not broken by the expansion of a foreign mineral inclusion; for if the cracks bounding the zircon in our specimen were to spread outwards all round to the diamond faces, it looks as though, from their position, neither of the two outer fragments, of the three into which

* In another, similar, case the cracks did reach the surface.

the diamond would break, would show signs of holes. Thus, even when a cleavage surface exhibits no imprint of a former mineral inclusion, it does not follow that the cleavage was not caused by an inclusion. It may be surmised, in passing, that the cracked yellow diamond spoken of in the last paragraph of Section 11 contained a mineral inclusion.*

The remarks in the present section apply in the main to more or less transparent diamond. Opaque diamond, other than bort, often enough is flawed, has little cohesion, and can be easily broken.

13. DIAMOND INCLUSIONS.

Diamonds enclosed in diamonds are not exactly uncommon. There are two very fine, nearly white, fair-shaped octahedra preserved in the De Beers office, each containing an inclusion. Both inclusions are practically of the same colour as their enclosures: one, a rounded octahedron, is certainly a diamond; the other is probably a diamond, but may be something else—say quartz as a remote possibility. These diamonds show very fine colour effects under the polariscope, but neither is much flawed internally. Where the strain comes from, though, when one diamond encloses another is an enigma.

Inclusions of diamond in diamond are not often symmetrical octahedra. One which was almost perfect in outline has been referred to in Section 4. A full description of this interesting object, which is still preserved, has been given by Williams (p. 506). The majority of such inclusions are irregular. A certain piece of Bultfontein cleavage, for example, which came from the wash already cracked nearly through, was found, when broken up, to contain a venerable-looking and ill-shapen flake of diamond, the cavity in which it had been being lined with a soapy-looking substance, possibly apophyllite. In this connection it is strange that the apophyllite, or whatever the substance may be, prefers to stick to the enclosure and not to the inclusion. Pretty often in cases of this sort the portions of diamond separated by the crack are of different quality and colour.

Tiny cavities, or what look like cavities, in diamonds are not uncommon. They may contain gas.¹ It is not the rule to find any large amount of mechanical strain in their vicinity.

14. DIAMONDS IN THE MATRIX.

Bauer tells us (p. 192) that different portions of one and the same broken diamond are never found lying close together in the matrix. He should have said that he had not heard of such a case. As a matter of fact a shattered diamond, mixed up with garnet and olivine, was extracted from its kimberlite matrix in the De Beers sorting office a few years ago. Furthermore, on two or three occasions it has happened that a diamond has

* But diamonds suffering strain, and with visible internal cracks, though apparently containing no inclusion, are sometimes met with.

been pieced together from its fragments found in the same wash, proving either that it had been broken in the machinery, or that the same fragments had come from somewhere close together in the pipe. And no doubt other fragments could be so pieced together if the sorters had nothing better to do than look for them. For another thing the finds of broken diamonds in the matrix have not been over many, much too few indeed to formulate general conclusions upon. Of the hundred or so specimens of diamond in the matrix which have reached the sorting office, the majority have been whole diamonds. More specimens could have been procured, no doubt, had they been sought for, especially in the hard blue ground lumps mined from the deeper levels of the De Beers or the Kimberley Mines; but a diamond in the matrix takes some finding, and a searcher might have to turn over many tons of rock before setting eyes on a diamond. At any rate, twenty is a liberal estimate of the number of broken diamonds found in their matrices that have been available for investigation; and there is no earthly reason why in some of these twenty cases we might not have found corresponding fragments of the diamonds could we have found corresponding halves of the matrices. Speaking of matrices, a fine lump of kimberlite, in dimensions about five inches long by four thick, picked up some years ago, contained two well formed diamonds, one a large yellow, the other a small brown one.

15. YELLOW DIAMONDS.

A word must be said about yellow diamonds. From the representative assortments of Pool and Dutoitspan diamonds, given near the beginning of this paper, we get the following comparisons:

	Pool. Per cent.	Dutoitspan. Per cent.
All stones	19.24	24.96
All cleavages	38.98	41.71
Rejection chips, rubbish, and bort	41.77	33.21

Thus there are twice as many cleavages as stones in the Pool, and about one and three-quarter times as many cleavages as stones in the Dutoitspan. Of these, yellow diamonds alone (including off-colours) are:

	Pool. Per cent.	Dutoitspan. Per cent.
Yellow stones	7.68	8.66
Yellow cleavages	11.04	9.93

and leaving out brown and "fancy" (which are largely brown) diamonds, which are:

	Pool. Per cent.	Dutoitspan. Per cent.
Fancy stones	—	3.01
Brown cleavages	5.79	7.88

we have left, of faintly tinted diamonds, whether spotted or not:

	Pool.	Dutoitspan.
	Per cent.	Per cent.
Stones other than brown or yellow	11.56	13.29
Cleavages other than brown or yellow	22.15	23.90

That is, the ratio of yellow cleavages to yellow stones is less than the corresponding ratios of brown or of colourless diamonds. It must have been on the basis of some such results as these that Bauer thought he had discovered that coloured diamonds resist fracture better than white ones. Pool information was probably all he had at his disposal at the time.

The significance of the above Pool and Dutoitspan statistics is to be gathered from the fact that foreign crystal inclusions in yellow diamonds are scarce. It is rare indeed for a yellow diamond to come in with a garnet or other mineral embedded in a broken face; and still more for such a mineral to be seen inside a whole stone. With regard to the latter contingency, however, it must be admitted that it would not always be easy to see an inclusion through the common sulcate faces of a yellow stone, though the former is evidence enough that foreign inclusions cannot be frequent. Again, South West African diamonds are often yellow, and the percentage of cleavages to stones is not great among them. An irregular yellow diamond, with grey corners, in my possession contains a garnet in a broken face. A piece of the garnet has been broken away exposing an excellent surface of attachment on the diamond. Spaces in the cavity between the diamond and the garnet are filled up here and there, as usual, with an opaque white substance.

The impression one gathers by way of inference from all these particulars, as well as from a scrutiny of the produce of the various mines round about Kimberley, is that nearly every yellow stone was formed rapidly in one piece out of a single bubble of liquid carbon (stained with iron oxide), or out of a bubble of some solvent containing carbon, thus occluding foreign solids; whereas the Wesselton or the Bultfontein octahedron was built up more slowly, step by step, and layer by layer, thus incorporating foreign solids in the diamond mass.¹ Naturally, a small, loose, foreign, solid body in a cavity larger than itself, occupied by a carbon bubble, would lie at the bottom of the hole if its specific gravity were greater than that of the bubble, or at the top if its specific gravity were less, and in either case need not be imprisoned in the final diamond crystal, though it might stick to a natural face of it. In the case of a diamond crystal continuing to grow by accretion of matter from adjacent carbon bubbles, the foreign solid might be imprisoned in the crystal, but not otherwise. Now clearly there would be more chance of this accretion in magmas where the bubbles were smaller and more numerous—as must have been the case for certain in the Wesselton and Bultfontein Mines,—than in magmas where the bubbles were larger and much further apart, as at Dutoitspan. In other words, assuming that

diamonds can grow if they get suitable opportunities; they got plenty of opportunities in Wesselton and Bultfontein, but not many in Dutoitspan. Thus, on this view, there is a relativity between

- (a) The numerous accretionary structures;
- (β) The frequent foreign inclusions;
- (γ) The numerous and small diamonds, of such a mine as Wesselton, and;
- (a') The few accretionary structures;
- (β') The rare foreign inclusions;
- (γ') The fewer and larger diamonds, of such a mine as Dutoitspan.

And this view is confirmed to my mind by such casual inspection as I have been able to make of diamonds won from mines situated outside the Kimberley district. In particular we may say that where diamonds are few or many in number, in any mine, there accretionary structures will be rare or numerous respectively. Wesselton diamonds are singularly accretionary.

How far apart from one another Kimberley diamonds *in situ* are may be illustrated by some examples. From May 16th to June 5th, 1917, eighteen working days, 129,640 loads, 2,074,240 cubic feet, of Dutoitspan blue ground were washed, yielding 20,957 carats of diamonds of all sizes, of which a first count gave 40,800 tiny diamonds averaging 78 to the carat (and only worth a penny a piece!), and 36,600 larger ones, of which one was of 126 carats and another of 174 carats; 77,400 diamonds in all, or one to every 27 cubic feet of broken blue ground, equivalent to one diamond to every 17 cubic feet of solid ground before mining. A similar count for Bultfontein showed 174,300 tiny diamonds and 207,900 larger ones, 382,200 in all, in 940,390 cubic feet of solid ground, or 1,504,624 cubic feet of broken ground, *i. e.* one diamond to every $2\frac{1}{2}$ cubic feet solid, or to every 4 cubic feet loose. Corresponding Wesselton results are 210,000 tiny diamonds, and 232,000 larger ones, 442,000 in all, in 1,717,730 cubic feet of solid ground, or 2,748,368 cubic feet of broken ground, *i. e.* one diamond to every 4 feet solid or to every $6\frac{1}{2}$ feet loose.

Before passing to other matters, it is worth while to suggest that the prevalent rounded form of yellow diamonds is not due, as has been suggested, to attrition or abrasion—the sulcate surfaces of these is suggestive of anything but that—but to the temperature at which they were crystallised and the influence of the colouring matter they contain.* Some such reason may account for the Pool or the Dutoitspan macles. A remote analogue is the fact that it is the presence or absence of magnesia which has determined

* "Rounded edges and other surface irregularities may, however, result from the corrosion of a crystal subsequent to its growth" (L. J. Spencer, Art. "Crystallography" in 'Ency. Brit.,' 11th ed., 1910). Surface abrasion is only shown to perfection on diamonds won from alluvial gravels.

whether the red crystal of the Burma ruby mines was to be the regular spinel or the rhombohedral ruby—remote, because in this case the magnesia is an important constituent of the spinel. A closer analogy is the prevailing prismatic habit of the (red) ruby as compared with the pyramidal (blue) sapphire; also aquamarine and emerald crystalline in somewhat different habits. On the other hand, variations in composition do not much affect the habits of garnet and tourmaline. Speaking at large, it may be said that among Kimberley diamonds form depends to a great extent upon colour. Thus, octahedra tend to be colourless; yellow diamonds are mostly rounded; bright yellow macles are decidedly uncommon, even in mines where yellow diamonds abound; ordinary grey bort, when it is symmetrical, is generally in cubes and spheres.

16. DIAMOND ENCLOSURES.

Just as foreign crystal inclusions are found in diamonds, so diamonds are found embedded in foreign crystals. It is the garnet that is chiefly seen associated with the diamond in this way. All Kimberley garnets are rolled or broken pieces, scarcely ever, if at all, possessing natural crystal faces. Generally also, though not invariably, the embedded diamond shows signs of wear and tear, proving that it must have had a history before the garnet formed upon it. Since, then, garnets are found in diamonds and diamonds in garnets, it follows that both must have originated in the same era of geological time and have crystallised under similar conditions.

Crystal diamond and bort are also found related to each other in a similar way.

17. YIELD.

The following table, taken from the last Annual Report of the De Beers Company, is interesting. It shows the yield in carats per hundred loads of the Wesselton and Bultfontein blue:

Table showing the Yield of Diamonds for Wesselton and Bultfontein.

For the 12 months ending June 30th.	Wesselton. (Carats.)	Bultfontein. (Carats.)
1898	27	—
1899	30	—
1900	30	—
1901	30	—
1902	30	21
1903	30	24
1904	28	29
1905	28	41
1906	28	36

For the 12 months ending June 30th.	Wesselton. (Carats.)	Bultfontein. (Carats.)
1907 .	32	32
1908 .	27	32
1909 .	34	38
1910 .	32	37
1911 .	27	38
1912 .	29	41
1913 .	27	42
1914 .	28	38
1915 .	26	35
1916 .	26	40

If the diamond content of the pipes be assumed constant at all depths, we should expect that better methods of extraction with progress of time, to give better yields. Since there are no signs of a better yield at Wesselton, the conclusion ought to be that the ground there gets slightly poorer with depth. At Bultfontein there would seem to have been an improvement with depth. At the latter mine there is a rough sort of correlation, accidental or not, between the yield and the quality of the diamonds given by it which may be expressed thus: When the yield of carats per hundred loads increases, the percentage of inferior diamonds tends to decrease; in other words, the diamonds are of the best quality where they are packed closest together. When Nature had an abundance of material she used it well in the interests of the market! This is for any given mine; it does not apply comparing one mine with another. For instance, Bultfontein has a higher yield than Dutoitspan, but not such good diamonds. Bultfontein blue ground is somewhat patchy; Wesselton, though poorer, is more uniform.

The De Beers and Kimberley yields are published together as one Annual values are given in the following table:

Table showing the Yield of Pool Diamonds.

For the 12 months ending June 30th.	Carats.
1892 . . .	92
1893 . . .	105
1894 . . .	89
1895 . . .	85
1896 . . .	91
1897 . . .	92
1898 . . .	80
1899 . . .	71
1900 . . .	67
1901 . . .	76

For the 12 months ending June 30th.	Carats.
1902	76
1903	61
1904	54
1905	46
1906	41
1907	37
1908	37
1909	42
1910	38
1911	28
1912	31
1913	29

Prior to the amalgamation of the various small companies working the De Beers and Kimberley Mines, the yield was considerably over 100 carats per 100 loads. Not only that, but many of the smaller diamonds used to be thrown away in the rough and ready methods of winning, leaving great heaps of tailings and *débris* well worth washing again in later years. Thus it would seem that the greatest wealth of these two mines was concentrated in the upper levels. On the other hand, only the richer areas were worked to any great extent in the earlier years, the poorer areas being left to spoil the averages of later times.

The Dutoitspan yield for the four years 1911-14 averaged about 22 carats per 100 loads.

18. ANOMALOUS BEHAVIOUR OF DIAMONDS ON THE GREASE TABLES.

Diamond appears to have a curious affinity for lime. It was discovered some years ago that diamonds from the Wesselton yellow ground were not readily caught on the grease separators, while it was exceptional for a diamond from the blue ground to fail to be caught. Experiments showed that yellow ground diamonds cleaned in acid lost their non-adhesive property and behaved normally on the tables like blue ground diamonds, whereas blue ground diamonds immersed for a few days in a paste of soft magnesian limestone, or of yellow ground, or in water that had been shaken up with lime or with yellow ground and dried, would not stick to the grease. Evidently the diamonds had acquired a thin coating of lime, which, when wetted, lessened the surface tension between them and the grease, and so caused them to behave like the ordinary gravel of the heavy pulsator deposit. Yellow ground, by the way, contains a large admixture of lime, some specimens losing a good half of their weight after treatment with aqua regia.

It is curious that diamonds left in the blue ground on the depositing floors for a number of years stick to the grease-tables less readily than those which are fresher from the mine. Some Bultfontein diamonds recently washed have this peculiarity. It is not unlikely that they have acquired a fine coating of lime from the soft limestone of the depositing floors.

19. ELECTRICAL CONDUCTIVITY.

True bort, and especially stewartite, is a good conductor of electricity. The regularly crystallised diamond is not, as is sometimes asserted, always a bad conductor. It is a bad conductor if its quality and texture be good, but if it be not highly transparent its electrical conductivity may be nearly as high as that of bort. Wesselton black bort, for instance, which is regularly crystallised though mostly in fragments, has a conductivity intermediate between transparent diamond and true bort. Some semi-transparent diamonds, greens and browns, are almost as good conductors as Wesselton bort. Further, the conductivity will pretty often have different values along different directions of the same stone. As a rule, the better the quality the lower the conductivity, one exception being that shot bort, though a worse conductor than ordinary grey bort, is a better than Wesselton black bort. We may arrange diamonds in order of conductivity, from the lowest to the highest, somewhat as follows :

- Fine transparent diamond.
- Inferior transparent diamond.
- Semi-transparent or translucent diamond.
- Wesselton black bort.
- Shot bort.
- Ordinary grey bort.
- Framesite (Transvaal black bort).
- Stewartite.

It would appear from this that impurities have as much to do with the electrical conductivity of diamonds as the habit has.

20. RELATIVE VALUES.

The relative average values of diamonds exceeding about .08 carat each in weight, from each mine, are approximately :

Dutoitspan	100
De Beers	86
Kimberley	76
Wesselton	68
Bultfontein	51

ADDENDA.

1. On the term "Macle" (see § 1).

According to English and French dictionaries, the word is generally said to be French and to be derived from Latin *macula* = a spot. Now it is true that most macles of diamond are spotted; but so are many well-shaped octahedra. Spots, indeed, are not by any means characteristic of macles, nor the most prominent feature. What is characteristic and prominent in macles is their shape and structure, so well expressed by the Dutch name "naadsteen" (seamstone). There is another French macle, namely, the water chestnut (*Trapa natans*), and in this case the origin of the word has not been traced. Was d'Isle thinking of the latter when he named the former?

2. A Variation in the Size of Wesselton Diamonds, with Depth of Working (see § 2).

It is curious that superimposed upon the irregular fluctuations of average size taking place from year to year, there appears to have been a definite increase in the average size of large Wesselton diamonds, *i. e.* in those of ten carats or more each. Also it takes a smaller yield now to produce a large diamond than it used to; and the ratio of the weight of large diamonds to the total yield has increased. This is best shown in a summary, *e. g.*,

Years.	Average size of large diamonds.	One large diamond in	Ratio of total weight of large diamonds to total yield.
1898-1906 . .	15.6 carats	769 carats	2.0 per cent.
1907-1917 . .	15.8 "	731 "	2.2 "

3. Cubes and Spheres (see § 6).

Cubes of translucent and of opaque bort are fairly common in Bultfontein, and of semi-translucent bort in Wesselton. Cubes of transparent diamond also occur, though rarely. The edges of the cubes are always much rounded. The indentations of the surfaces of some cubes tend to a square outline, giving an impression that the mass is built up of a multitude of tiny cubes, whereas the indentations of the surfaces of octahedra are mostly equilateral triangles in plan. Many cubes, as well as octahedra, are pitted with rounded holes (*cf.* § 12). An existing small yellow cube of transparent diamond shows internal strain.

Spheres of translucent bort are called shot bort. Apparently the crystalline aggregates in these incline to a radial arrangement about a central nucleus which in most cases revealed by cleavage, if not all, consists of a discrete core of diamond or bort.

NOTE ON THE RESOLVABILITY OF THE MINORS OF A COMPOUND DETERMINANT.

By SIR THOMAS MUIR, LL.D.

(1) The theorem which is usually made the basis of investigation on this subject is that exemplified by Spottiswoode* in 1853 and formally enunciated and established by Franke and Borchardt† in 1862. It is in effect that *any k-line minor of the mth compound of $|a_{1n}|$ is equal to*

$$K \cdot |a_{1n}|^{k - (n-1)m}.$$

K being the complementary of the corresponding minor in the $(n-m)^{th}$ compound. The theorem is of course directly useful for the end in view when $k > (n-1)m$; in other cases it may turn out indirectly useful, but only because the said corresponding minor occasionally lends itself more readily to decomposition than the minor actually set.

(2) There is one case, however, in which it is of no avail at all; namely, where

$$k = (n-1)m \text{ and } n = 2m,$$

for then it takes an illusory form, all the information given by it being that the minor in question is equal to itself. The existence of this case makes the general problem difficult of treatment; and it is consequently desirable for future work to know how the question of resolvability stands in regard to it. The instance of it where

$$n = 4, \quad m = 2, \quad k = 3,$$

is the simplest, but, as there are then 400 minors to be adjudicated on, considerable instruction necessarily results from examination of it. If the basic determinant be taken in the form

$$|a_1 b_2 c_3 d_4|$$

the 400 in question are the three-line minors of

$$\begin{vmatrix} |a_1 b_2| & |a_1 c_2| & \dots & |c_1 d_2| \\ |a_1 b_3| & |a_1 c_3| & \dots & |c_1 d_3| \\ \cdot & \cdot & \cdot & \cdot \\ |a_3 b_4| & |a_3 c_4| & \dots & |c_3 d_4| \end{vmatrix}.$$

* 'Crelle's Journ.,' li, pp. 366-368.

† 'Crelle's Journ.,' lxi, pp. 350-358.

(3) The most numerous and the most interesting of them are those that are *irresolvable*. As a typical example we may take

$$\begin{vmatrix} 125 \\ 125 \end{vmatrix} \text{ i. e. } \begin{vmatrix} |a_1b_2| & |a_1b_3| & |a_2b_4| \\ |a_1c_2| & |a_1c_3| & |a_2c_4| \\ |b_1d_2| & |b_1d_3| & |b_2d_4| \end{vmatrix},$$

the irresolvability of which is implicated in the equality

$$|a_1b_2| |a_1c_3| |b_2d_4| = |a_1b_2c_4| |a_1b_3d_3| + a_1b_3|a_1b_2c_3d_4|. \quad (\text{I})$$

One way of proving this may be put shortly as follows. In the given determinant

$$\text{the cofactor of } |b_2d_4| = a_1|a_1b_2c_3|,$$

$$\text{the cofactor of } |a_2c_4| = b_1|a_1a_2d_3|,$$

$$\text{and the cofactor of } |a_2b_4| = a_1|b_1c_2d_3| + c_1|a_1b_2d_3|*,$$

so that the determinant

$$= |a_1b_2d_3| \{c_1|a_2b_4| - b_1|a_2c_4|\} \\ + a_1 \{ |b_2d_4| |a_1b_2c_3| + |a_2b_4| |b_1c_2d_3| \}.$$

By increasing the first part of this by

$$|a_1b_2d_3| \cdot a_1 \cdot |b_2c_4|$$

and diminishing the second by the same, the former becomes

$$|a_1b_2d_3| |a_1b_2c_4|,$$

and the latter becomes

$$a_1 \{ |b_2d_4| |b_2a_1c_3| - |b_2a_4| |b_2d_1c_3| - |b_2c_4| |b_2a_1d_3| \}$$

which is readily seen to be equal to

$$a_1b_2|b_2d_4a_1c_3|,$$

as desired.

(4) We may note in passing that to the development thus obtained,

$$|a_1b_2c_4| |a_1b_2d_3| + a_1b_2|a_1b_2c_3d_4|$$

there is an alternative form, namely,

$$|a_1b_2c_3| |a_1b_2d_4| + a_2b_1|a_1b_2c_3d_4|.$$

The equivalence of the two is established by noting that their difference

$$|a_1b_2c_4| |a_1b_2d_3| - |a_1b_2c_3| |a_1b_2d_4| + |a_1b_3| |a_1b_2c_3d_4|$$

is the extensional of

$$c_4d_3 - c_3d_4 + |c_3d_4|$$

which manifestly vanishes.

* This identity used by Binet in 1812: see 'Hist.' i, p. 89.

(5) The distinguishing feature of the equality established in § 3 is the prominence held in it by the collocation of elements a_1b_2 ; and we see that to every such collocation, thirty-six in all, there corresponds a like equality having on its left an irresolvable minor. In this connection the next point is to ascertain whether there be not other irresolvable minors in which a_1b_2 plays the same part. Observing the positions occupied in

$$\begin{vmatrix} a_1 & a_2 & a_3 & a_4 \\ b_1 & b_2 & b_3 & b_4 \\ c_1 & c_2 & c_3 & c_4 \\ d_1 & d_2 & d_3 & d_4 \end{vmatrix}$$

by the minors involved in the equality (I) we see that while retaining a_1b_2 unchanged in all its positions we may make (α) interchange of the third row with the fourth, (β) interchange of the third column with the fourth, and (γ) both of these interchanges simultaneously. Doing this we obtain

$$(\alpha) \quad \begin{vmatrix} a_1b_2 & a_1d_3 & b_3c_4 \end{vmatrix} = a_1b_2c_3 |a_1b_2d_4| + a_1b_2 |a_1b_2d_3c_4|$$

i. e. $\begin{vmatrix} 125 \\ 134 \end{vmatrix} = |a_1b_2c_3| |a_1b_2d_4| - a_1b_2 |a_1b_2c_3d_4|.$

$$(\beta) \quad \begin{vmatrix} a_1b_2 & a_1c_4 & b_2d_3 \end{vmatrix} = |a_1b_2c_3| |a_1b_2d_4| + a_1b_2 |a_1b_2c_4d_3|$$

i. e. $\begin{vmatrix} 134 \\ 125 \end{vmatrix} = |a_1b_2c_3| |a_1b_2d_4| - a_1b_2 |a_1b_2c_3d_4|.$

$$(\gamma) \quad \begin{vmatrix} a_1b_2 & a_1d_4 & b_2c_3 \end{vmatrix} = |a_1b_2d_3| |a_1b_2c_4| + a_1b_2 |a_1b_2d_4c_3|$$

i. e. $\begin{vmatrix} 134 \\ 134 \end{vmatrix} = |a_1b_2c_4| |a_1b_2d_3| + a_1b_2 |a_1b_2c_3d_4|.$

There are thus *four* different irresolvable minors marked out by the collocation a_1b_2 , the set of four being most suitably arranged thus :

$$\begin{vmatrix} 125 \\ 125 \end{vmatrix} = |a_1b_2c_4| |a_1b_2d_3| + a_1b_2 |a_1b_2c_3d_4| = \begin{vmatrix} 134 \\ 134 \end{vmatrix}$$

$$\begin{vmatrix} 125 \\ 134 \end{vmatrix} = |a_1b_2c_3| |a_1b_2d_4| - a_1b_2 |a_1b_2c_3d_4| = \begin{vmatrix} 134 \\ 125 \end{vmatrix}.$$

As a consequence we know of the existence of 144 irresolvables among the 400 three-line minors of the second compound of $|a_1b_2c_3d_4|$; also that the 144 may be arranged in pairs the members of which are equal.

(6) In turning to resolvables it is convenient to deal first with an equality immediately derivable from (I), namely,

$$\begin{vmatrix} |a_1b_2| & |a_1b_3| & |a_2b_3| \\ |a_1c_2| & |a_1c_3| & |a_2c_3| \\ |b_1d_2| & |b_1d_3| & |b_2c_3| \end{vmatrix} = |a_1b_2c_3| |a_1b_2d_3|,$$

got by changing the suffix 4 everywhere into 2. It is seen to concern the elements of a 4 - by - 3 array, and to have its left member conveniently symbolisable by

$$\left\{ \begin{array}{l} a_1 \ a_2 \ a_3 \\ b_1 \ b_2 \ b_3 \\ c_1 \ c_2 \ c_3 \\ d_1 \ d_2 \ d_3 \end{array} \right.$$

where the first two rows have a double link of connection and the other two a single link. With the first two still doubly connected, but with different single ends, there is the diagram

$$\left\{ \begin{array}{l} a_1 \ a_2 \ a_3 \\ b_1 \ b_2 \ b_3 \\ c_1 \ c_2 \ c_3 \\ d_1 \ d_2 \ d_3 \end{array} \right.$$

giving us the equality

$$\left| \begin{array}{ccc} |a_1 b_2| & |a_1 b_3| & |a_2 b_3| \\ |a_1 d_2| & |a_1 d_3| & |a_2 d_3| \\ |b_1 c_2| & |b_1 c_3| & |b_2 c_3| \end{array} \right| = |a_1 b_2 c_3| |a_1 b_2 d_3|,$$

so that we have

$$\frac{124}{125} = |a_1 b_2 c_3| |a_1 b_2 d_3| = \frac{124}{134}.$$

As the double-ends might also be at $a_1 c_1$, $a_1 d_1$, $b_1 c_1$, $b_1 d_1$, $c_1 d_1$; and as the number of 4-by-3 arrays is 4; and as there is as many 3-by-4 arrays equally fruitful; the full number of three-line minors resolvable in this particular way is

$$2 \times 6 \times 4 \times 2 \quad \text{i.e. } 96.$$

(7) Changing the d 's of the foregoing into c 's, we obtain the familiar equality regarding the adjugate of $|a_1 b_2 c_3|$

$$\left\{ \begin{array}{l} a_1 \ a_2 \ a_3 \\ b_1 \ b_2 \ b_3 \\ c_1 \ c_2 \ c_3 \end{array} \right\} \equiv \left| \begin{array}{ccc} |a_1 b_2| & |a_1 b_3| & |a_2 b_3| \\ |a_1 c_2| & |a_1 c_3| & |a_2 c_3| \\ |b_1 c_2| & |b_1 c_3| & |b_2 c_3| \end{array} \right| = |a_1 b_2 c_3|^2$$

Of this there must be sixteen instances, one corresponding to every primary minor of $|a_1 b_2 c_3 d_4|$.

(8) An example of the next type of resolvable minors is $\frac{123}{125}$, the equality connected with which is

$$\begin{vmatrix} |a_1b_2| & |a_1b_3| & |a_1b_4| \\ |a_1c_2| & |a_1c_3| & |a_1c_4| \\ |b_1d_2| & |b_1d_3| & |b_1d_4| \end{vmatrix} = a_1b_1|a_1b_2c_3d_4|$$

and is established in the same way as that of § 3. The left-hand member being symbolisable by

$$\left[\begin{array}{c} \left\{ \begin{array}{c} a_1 \\ b_1 \\ c_1 \\ d_1 \end{array} \right\} \begin{array}{ccc} a_2 & a_3 & a_4 \\ b_2 & b_3 & b_4 \\ c_2 & c_3 & c_4 \\ d_2 & d_3 & d_4 \end{array} \end{array} \right]$$

we may reason in regard to it exactly after the manner of § 6, obtaining first the double result

$$\begin{vmatrix} 123 \\ 125 \end{vmatrix} = a_1b_2|a_1b_2c_3d_4| = \begin{vmatrix} 123 \\ 143 \end{vmatrix}$$

and thereafter the number of such resolvables to be 96.

(9) Less general than the immediately preceding, although not thence obtainable by substitution, is the equality

$$\begin{vmatrix} 123 \\ 123 \end{vmatrix} = \begin{vmatrix} |a_1b_2| & |a_1b_3| & |a_1b_4| \\ |a_1c_2| & |a_1c_3| & |a_1c_4| \\ |a_1d_2| & |a_1d_3| & |a_1d_4| \end{vmatrix} = a_1^2|a_1b_2c_3d_4|.$$

Here, as in § 7, the change of rows into columns is fruitless, the left-hand member being symbolisable both by

$$\left[\left(\begin{array}{c} \left\{ \begin{array}{c} a_1 \\ b_1 \\ c_1 \\ d_1 \end{array} \right\} \begin{array}{ccc} a_2 & a_3 & a_4 \\ b_2 & b_3 & b_4 \\ c_2 & c_3 & c_4 \\ d_2 & d_3 & d_4 \end{array} \right) \right] \text{ and } \left[\left(\begin{array}{c} \left\{ \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} \right\} \begin{array}{ccc} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{array} \right) \right]$$

The number of minors so resolvable is manifestly sixteen.

(10) Lastly, by returning to § 8 and changing the d 's into c 's we obtain

$$\begin{vmatrix} 123 \\ 124 \end{vmatrix} = \begin{vmatrix} |a_1b_2| & |a_1b_3| & |a_1b_4| \\ |a_1c_2| & |a_1c_3| & |a_1c_4| \\ |b_1c_2| & |b_1c_3| & |b_1c_4| \end{vmatrix} = 0.$$

The construction-symbol for the determinant is either

$$\left[\left(\begin{array}{c} \left\{ \begin{array}{c} a_1 \\ b_1 \\ c_1 \end{array} \right\} \begin{array}{ccc} a_2 & a_3 & a_4 \\ b_2 & b_3 & b_4 \\ c_2 & c_3 & c_4 \end{array} \right) \right] \text{ or } \left[\left(\begin{array}{c} \left\{ \begin{array}{c} a_1 \\ a_2 \\ a_3 \\ a_4 \end{array} \right\} \begin{array}{ccc} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ d_1 & d_2 & d_3 \end{array} \right) \right]$$

the former showing affinity with the symbol of § 8 and the latter with that of § 6. Using either symbol, we readily infer that the number of vanishing minors is thirty-two.

(11) There thus results the following census :

Irresolvable	144
Resolvable, type 33	96
„ type 114	96
„ type 3 ³	16
„ type 1 ²⁴	16
Vanishing	32

giving the requisite total of 400. As in the first three groups each minor has an equivalent companion, there are altogether 168 such pairs of equals

CAPETOWN, SOUTH AFRICA,
April 11th, 1917.

A LUNAR PERIOD IN THE RATES OF EVAPORATION AND RAINFALL?

By J. R. SUTTON, ScD., Hon. Memb. R. Met. S.

Three factors are commonly regarded as fundamental to evaporation, namely, the temperature of the water, the humidity of the air, and the velocity of the wind.

Actually, only the first of these three is fundamental. The second is subsidiary, acting to check the rate at which the evaporation goes on. The third is a perturbation pure and simple; its only function is to replace the damp air over an evaporating surface by other air; so that if the other air be damper, or less damp, than the replaced air, the evaporation will be checked, or accelerated, accordingly.

The present paper is intended to call attention to the possibility of another fundamental factor, not, apparently, hitherto suspected; and which, although feeble, may have an important bearing on the kinetic theory.

Briefly put, the object of the investigation here to be described was to determine whether a variation of gravity was directly or indirectly accompanied by any appreciable variation of the motion of the molecules concerned in the evaporation from a water surface. If there be such a relation, it cannot be very prominent, and only to be discovered in the results of long and careful observation. The existence and magnitude of a tiny lunar atmospheric tide has long been known; and because of this tide the barometer rises and falls slightly twice a day; the corresponding maxima of pressure occurring when the moon is near the meridian, above or below the horizon. There is doubtless, also, a related very small variation in the velocity of the wind the magnitude of which has still to be measured. Consequently, we should expect, to begin with, that if the moon has any influence at all upon the rate of evaporation, it would be exerted by means of the agencies of pressure and wind variation—both too minute, it would seem, to be of much account.

However that may be, the results of the observations with the evaporating tank at Kenilworth have been examined for the purpose of ascertaining whether there is any lunar effect at all of any sort.

A description of the tank, and the way it is used, will be found in the

TABLE I.
Hourly Evaporation arranged according to the Solar Day.

M.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	M.	XIII.	XIV.	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.	XXII.	XXIII.	XXIV.	XXV.	Total.	
March 1	017	017	009	011	005	010	017	012	010	015	008	012	016	019	020	019	017	023	011	012	008	012	013	010	010	010	323 in.
2	10	5	8	6	2	6	10	8	6	4	8	11	8	13	9	15	10	15	10	5	6	2	4	9	190	29	
3	5	8	6	6	5	4	7	7	8	6	10	17	11	23	21	19	21	16	9	3	8	5	5	8	238	33	
4	6	6	9	9	6	4	7	5	13	7	11	17	12	20	20	20	18	10	3	7	6	6	9	7	238	35	
5	10	9	5	8	7	5	11	7	7	10	13	12	20	20	21	19	22	12	7	8	3	8	8	6	258	37	

TABLE II.
Hourly Evaporation arranged according to the Lunar Day.

	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.	XIV.	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.	XXII.	XXIII.	XXIV.	XXV.
March 1	020	017	017	009	011	005	010	017	012	010	015	008	012	016	019	020	019	017	023	011	012	008	012	013	010
" 2	10	5	8	6	2	6	10	8	6	4	8	11	8	13	9	15	10	15	10	5	6	2	4	9	5
" 3	5	8	6	6	5	4	7	7	8	6	10	17	11	23	21	19	21	16	9	3	8	5	5	8	6
" 4	6	6	9	9	6	4	7	5	13	7	11	17	12	20	20	18	10	3	7	6	6	9	7	10	9
" 5	5	8	7	5	11	7	7	10	13	12	20	20	21	19	22	12	7	8	3	8	8	6	9	9	8

Report of the Cape Meteorological Commission for 1902. Hourly observations of evaporation from the surface of this tank are available since 1899. The measures are taken to the third decimal place in inches, and averaged for each hour of each month and year.

The great labour in the present case was the clerical one of rearranging the tabular measures of evaporation, so that they should conform to the lunar day instead of to the solar day. This was done by tabulating the hourly quantities of evaporation in twenty-five columns, the thirteenth, or central, column containing the amount of evaporation in the hour during which the moon crossed the meridian above the horizon (lunar noon); the mean of the first and twenty-fifth columns being regarded as the evaporation in the hour during which the moon crossed the meridian below the horizon (lunar midnight). One hundred and twenty lunar months, from August, 1899, to April, 1909, have been dealt with in this way and averaged.

Tables I and II will give an idea of the method of procedure. Table I is an extract from the record books, and shows the hourly evaporation for the five days, March 1st to 5th, 1908. On these days the moon made its upper meridian passage on—

- March 1.—Between 11 a.m. and noon.
- „ 2.—About noon.
- „ 3.—Between noon and 1 p.m.
- „ 4.—Between 1 and 2 p.m.
- „ 5.—Between 2 and 3 p.m.

And in Table I the quantities of evaporation for these particular hours is put in *italic type*. The rearrangement in lunar terms is given in Table II.

The averages deduced from observations made on 3408 lunar “days,” for each hour of the lunar “day,” are given in Table III:

TABLE III.

Hourly Mean Evaporation arranged according to the Lunar Day.

Hour.	Inch.	Hour.	Inch.	Hour.	Inch.
I	·0070	X	·0069	XIX	·0070
II	69	XI	69	XX	71
III	69	XII	70	XXI	70
IV	70	XIII	69	XXII	70
V	70	XIV	69	XXIII	71
VI	69	XV	70	XXIV	70
VII	69	XVI	70	XXV	71
VIII	69	XVII	70		
IX	69	XVIII	71	Mean	·0070

Excepting that the values in the second half of the lunar day are somewhat greater than those in the first, there is nothing in Table III to suggest outside influences. In particular there is nothing in the table that would indicate a lunar influence. We can, however, as it happens, show how such variation as there is in the hourly rate arises:

First let us compare the evaporation on dry days with that on wet days.

The evaporation, smoothed in threes, on dry days is given in Table IV:

TABLE IV.

Hourly Mean Evaporation on Dry Days, arranged according to the Lunar Day.

Hour.	Inch.	Hour.	Inch.	Hour.	Inch.
I .	·0074	X .	·0075	XIX .	·0076
II .	73	XI .	75	XX .	75
III .	73	XII .	76	XXI .	75
IV .	73	XIII .	76	XXII .	75
V .	74	XIV .	76	XXIII .	75
VI .	74	XV .	76	XXIV .	75
VII .	74	XVI .	76	XXV .	74
VIII .	74	XVII .	76		
IX .	74	XVIII .	76		

In Table IV, by "dry days" is meant all those days upon which there was no rain at all, or none sufficient to materially disturb the ordinary run of the evaporation. In a few instances, however, when there have been short passing showers yielding no great quantity of rain, but where the weather has made itself felt, a small plus correction has been necessary to the observed quantities of evaporation, so as to bring them into line with the normal diurnal curves of evaporation for the months in which they occur. This correction has in all cases been made with due caution. Table IV gives a definite curve with a minimum at the second and third hours and a maximum at the fifteenth, with a mean daily range, $M - m$, of ·0003 inch, corresponding to a total range, $M - m$, of nearly an inch in the whole 2878 days accounted for in the table. If we could regard the numbers shown in Table IV as a definite lunar effect, we should have to admit an effect so great as to give a range of upwards of 4 per cent. of the hourly mean rate of evaporation. Which is to say that the evaporation is enlarged or diminished by 2 per cent., according as the moon near the meridian is respectively above or below the horizon. This amount of deviation is undoubtedly greater than could have been reasonably expected.

Upon reflection, however, it will be obvious that no matter what the arrangement (lunar, solar, or what not) of the hourly quantities of evaporation, the effect of eliminating wet days from the account will be to give a curve for the dry days with a maximum near the central ordinate, and a

minimum at each end. For, after a wet day, as the air dries, the evaporation will frequently be slow at first and will increase more or less gradually afterwards. Also the approach of a rainy day will often enough be preceded by a moister air which will diminish the evaporation, unless an increased wind velocity counteracts the humidity.

The mean hourly, unsmoothed, evaporation for the 530 rainy days omitted in making Table IV is given in Table V.

TABLE V.
Hourly Mean Evaporation on Rainy Days, arranged according to the Lunar Day.

Hour.	Inch.	Hour.	Inch.	Hour.	Inch.
I .	.0066	X .	.0052	XIX .	.0057
II .	63	XI .	49	XX .	59
III .	62	XII .	51	XXI .	59
IV .	60	XIII .	48	XXII .	59
V .	61	XIV .	47	XXIII .	60
VI .	59	XV .	49	XXIV .	59
VII .	59	XVI .	49	XXV .	59
VIII .	56	XVII .	52		
IX .	54	XVIII .	53		

The values shown in Table V are surprising. There would be no apparent *a priori* reason why, since rain is to be expected at any hour of the lunar day, there should be more evaporation in any one hour than in another. Still less that an arrangement of the observed evaporation in any series of hours other than twenty-four, should give a fairly symmetrical curve with a definite minimum near the central ordinate. Moreover, with a range equivalent to 34 per cent. of the mean value.

TABLE VI.
Total Hourly Rainfall in 120 Months arranged according to the Lunar Day.

Hour.	Inch.	Frequency.	Hour.	Inch.	Frequency.	Hour.	Inch.	Frequency.
I .	6.876	123	X .	8.347	138	XIX .	7.638	122
II .	6.448	124	XI .	5.841	142	XX .	5.758	122
III .	9.065	131	XII .	8.798	143	XXI .	6.288	118
IV .	8.960	135	XIII .	7.606	143	XXII .	4.188	121
V .	6.765	141	XIV .	8.796	137	XXIII .	6.305	121
VI .	9.366	143	XV .	5.770	131	XXIV .	7.055	127
VII .	8.919	147	XVI .	6.180	131	XXV .	6.342	123
VIII .	7.446	143	XVII .	5.370	128			
IX .	5.721	142	XVIII .	7.481	128	Totals	177.329	3304

Let us inquire, therefore, whether there has been an unequal distribution of rainfall in the whole 120 lunar months. Table VI gives the total rainfall, and the total number of hours of rain, deduced from the anaxanometer

records, for every shower in the whole period exceeding $\cdot 001$ inch an hour. It summarises, therefore, rather more material than is used in Table V. The hourly frequencies are smoothed in threes, but the quantities are unsmoothed.

We find in Table VI what we should expect, namely, a considerable variation in the quantity of rain, hour by hour; and what we should not expect, namely, a definite lunar curve of frequency with a maximum near moon rise and a minimum just after moon set. Quantities and frequencies are distributed as follows (neglecting the meridiem hour):

	Inches.	Frequency.
Ante-meridiem	52.2 per cent.	50.0 per cent.
Post-meridiem	43.5 "	45.7 "

Thus, underlying the irregularity of the quantities there is a greater tendency to an ante-meridiem maximum than is shown by the more regular frequencies. In fact the hourly intensities of rainfall are:

Ante-meridiem	$\cdot 056$ inch
Post-meridiem	$\cdot 051$ "

Hence it is evident that the vagueness of Table III is due to the peculiar incidence of rainfall. Rain coming most often, as it does, about the time of moon rise will naturally cause the evaporation to be less in the hours following that time than in the hours preceding. And so we get an explanation of the run of the hourly rates of Table V; and these just happen to nearly counteract the run of the hourly rates of Table IV.

If now we introduce corrections into Table III on the assumption that the mean evaporation curve for rainy days ought to be a straight line, we get the unsmoothed values of Table VII.

TABLE VII.
Corrected Hourly Mean Evaporation arranged according to the Lunar Day.

Hour.	Inch.	Hour.	Inch.	Hour.	Inch.
I .	$\cdot 0069$	X .	$\cdot 0069$	XIX .	$\cdot 0070$
II .	68	XI .	70	XX .	71
III .	68	XII .	71	XXI .	70
IV .	69	XIII .	71	XXII .	70
V .	69	XIV .	71	XXIII .	70
VI .	69	XV .	71	XXIV .	69
VII .	69	XVI .	71	XXV .	70
VIII .	69	XVII .	71		
IX .	69	XVIII .	71		

The curve given by Table VII is in substantial agreement with Table IV, and considering that it is unsmoothed in any way is remarkably regular.

Judging by the magnitude of the hourly variation, by the time at which the evaporation is greatest and least, and by the fact that there is only one maximum and minimum in the day, it is evident that the curve is not dependent upon the lunar atmospheric tide nor upon whatever small variation of wind velocity is generated by the tide.

No useful purpose would be served by speculating further at this stage on the results arrived at in Tables VI and VII. For the present they can only be regarded as provisional. But it would be well if they could be tested by means of observations in some rainless area. Meanwhile I have begun the investigation of a second period of 120 months' observations, albeit without any great expectation of completing it within any short time to come.

COLOUR AND CHEMICAL CONSTITUTION.

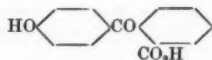
PART II.—THE SPECTRA OF THE MIXED PHTHALEINS AND OF THE
SULPHONE-PHTHALEINS.

BY JAMES MOIR, M.A., D.Sc., F.I.C.

In the first part of this paper (read March 21, 1917; sent in January 18th) an attempt was made to discover the laws which govern the colour of organic dye-stuffs by choosing one of such simple constitution and easy manufacture that it was not a troublesome matter to prepare and examine a large number (over fifty) of derivatives of the parent substance, and thus solve, to a large extent, the problem of the effect on colour of varying the substituting groups—without wasting a lifetime in the preparation of the necessary compounds.

Phenolphthalein is a quasi-symmetrical compound, since it contains two equal $-C_6H_4OH$ groups, and the same is true of about forty of the compounds described in Part I; but in addition a number of compounds were described which have $-C_6H_5$ plus $-C_6H_4OH$, and are thus mixed phthaleins of a kind, although not dibasic like the true phthaleins. They were made by condensing benzoylbenzoic acid with phenols.

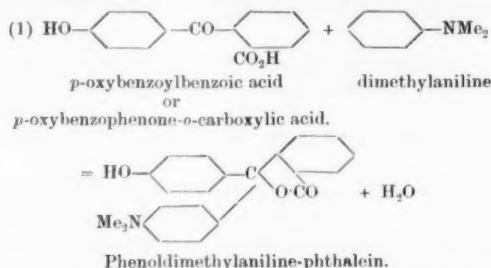
In April of this year (1917) it struck me that substituted benzoyl-benzoic acids, particularly



(which is easily made from phenolphthaleinoxime), might be used to make true dibasic mixed phthaleins containing one $-C_6H_4OH$ group as well as the residue of another phenol $-C_6H_4-X.OH$.

On trying the experiments I found that the above-mentioned *p*-oxy-benzoyl-*o*-benzoic acid is capable of combining with all sorts of phenols with the utmost ease; in most cases simple boiling of a mixture of the ingredients without a condensing agent for ten seconds suffices to give a good yield of the desired mixed phthalein. In addition, the acid can combine with the most unexpected aromatic substances (phenol-esters, anhydrides, and amines, whether free or substituted in the nitrogen) to

yield mixed phthaleins of a new type, although phthalic anhydride does not condense with these substances. Thus coumarine, quinosol, dimethylaniline, and to some extent even menthol, yield phthaleins with this acid; in fact, probably any ring compound with $-\text{OH}$ or $-\text{NH}_2$, either free or substituted, will react in this way, provided that it has a free para- or ortho-position. The following scheme represents the combination in the simpler of the two cases, the para-position being the preferred one:



Unfortunately for my priority, the principle of this synthesis has in the meantime been discovered independently in the United States, Messrs. Orndorff and Murray ('Journ. Amer. Chem. Soc.' April, 1917) having published the preparation of several of my mixed phthaleins and thus evidently anticipated me by a few months. They did not, however, "spectroscopically" their compounds.

Table I gives the results of my investigation.

It may be noted that (1) phenolcoumarinphthalein is a direct derivative of ordinary phenolphthalein, being its *o*-acrylic acid compound, the acrylic group $-\text{CH}:\text{CH}\cdot\text{CO}_2\text{H}$ having practically the same effect on the colour as a methyl group; (2) quinosol is 8-oxyquinoline, *i.e.* with the $-\text{OH}$ group in a different ring from the nitrogen; the pyridine ring has less effect on the colour than the benzene-ring, comparing the quinosol- with the α -naphthol- compound; (3) the hydrogenated rings of menthol and tetrahydronaphthol have much less effect than the corresponding thymol and naphthol rings, but owing to one attachment of the tetrahydronaphthol ring being *meta* to the $-\text{OH}$ (see Part I), the effect is greater than that of a powerful group (*e.g.* $-\text{OCH}_3$) in the *ortho*-position; (4) the effect of the phenolic $-\text{OH}$ having to take up the *ortho*-instead of the usual *para*-position to the central carbon is a greatly increased shift of the colour or lowering of frequency, *e.g.* the β -naphthol derivative.

It is probable that the absorption-band of di- β -naphtholphthalein (from β -naphthol and phthalic anhydride) is right off the visible spectrum, which accounts for it not having been recognised.

The mixed aminophthaleins are remarkably sensitive to excess alkali,

which easily bleaches their colour; otherwise from their brilliant colours they would make excellent indicators.

TABLE I.
Spectra of the Dibasic Mixed Phthaleins.

No.	Name.	Colour in Alkali.	Wave-length of Absorption Band.	Percentage Difference from Phenolphthalein.	Wave-length in conc. H_2SO_4 .
1	Phenolthymolphthalein	Purple	578	4.1	About 520
2	Phenolorthocresolphthalein	Pink	563	1.5	
3	Phenolguaiacolphthalein	Purple	580	4.4	About 525
4	Phenolcarvacrolphthalein	"	580	4.4	
5	Phenol- α -naphtholphthalein	Indigo	607*	8.7	566 and violet.
6	Phenol- β -naphtholphthalein	Green	637*	13.0	
7	Phenolresorcinolphthalein (mixture of two)	Dichroic-red	558 and 494	Abnormal	503
8	Phenolphloroglucinolphthalein (mixture of two)	Dichroic-red	550 and 490	"	nil.
9	Phenolpyrogallolphthalein (mixture of two)	Dichroic-red	580 and 480	"	"
10	Phenolpyrocatecholphthalein	Purple	579	4.3	522
11	Phenolcoumarinphthalein	Pink	565 (slight fluorescence)	1.9	
12	Phenolquinosolphthalein	Blue	598	7.3	520
13	Phenolmentholphthalein	Pink	About 563	1½	
14	Phenoltetrahydro- α -naphtholphthalein	Blue-purple	588	5.8	530
15	Phenolanilinephthalein	Pink	558	0.7	Ultra-violet.
16	Phenolmonomethylanilinephthalein	Red-purple	576	3.8	"
17	Phenoldimethylanilinephthalein	Blue-purple	587½	5.6	"
18	Phenol- α -naphthylaminephthalein	Yellow-green	About 740* (fluorescent)	25	

* Plus ultra-violet.

In any case the easy reaction of all these organic compounds with oxybenzoylbenzoic acid affords a simple analytical test for phenols and amines, the position of the absorption band in the spectrum at once indicating what the phenol (or amine) used was.

If a numerical comparison be made between the spectra of any one of these mixed phthaleins and the corresponding ordinary phthalein, the remarkable discovery is made that the mixed phthalein spectrum is not quite intermediate between those of phenolphthalein and the other phthalein. Thus di-phenolphthalein (ordinary phenolphthalein) has λ 554, phenolthymolphthalein has λ 578, and di-thymolphthalein has λ 597. In other words, the entrance of one pair of methyl and isopropyl groups depresses the

frequency of vibration by 4.1 per cent., and the entrance of a second pair only brings the depression to 7.2 per cent. The same law is true of all the others, and may be stated as follows—that the entrance of the first grouping causes a change equal to about 58 per cent. of the total change due to two groupings.

This phenomenon is easily explained in terms of the interference of the rings, or rather of their projecting substituents, with one another (see Part I). It is obvious that the entrance of one $-\text{OCH}_3$ group into one ring of phenolphthalein will cause a great interference with the other empty ring and that the entrance of $-\text{OCH}_3$ into the other ring will cause a second interference, but not so great.

THE SULPHONE-PHTHALEINS (SO-CALLED SULPHUREINS).

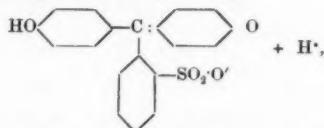
I have discovered that some of these can be easily made by heating phenols with "saccharin" (the artificial sweetening agent) and a little sulphuric acid, followed by boiling the product with dilute acid to split off ammonia. They form excellent indicators. In my opinion, phenolsulphonephthalein is the best general indicator yet discovered for sensitiveness and definiteness of colour. Its constitution is that of phenolphthalein with the group CO replaced by SO_2 ; it is therefore, chemically speaking, the ortho-sulphonic acid of benzaurine (see Part I). The following table gives the spectra of those examined so far.

TABLE II.

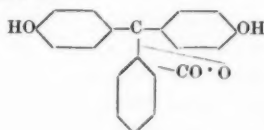
No.	Name.	Colour in Alkali.	Wave-length absorbed in Alkali.	Wave-length absorbed in N HCl .	Wave-length absorbed in conc. H_2SO_4 .
1	Phenolsulphonephthalein .	Purple-pink	563	507	500 (practically same as phenolphthalein).
2	Thymolsulphonephthalein	Blue	604	?	About 570
3	α -naphtholsulphonephthalein	Olive-green	About 730	?	About 720 and 500
4	Guaiacolsulphonephthalein	Blue	608	565	570
5	Resorcinolsulphonephthalein	Salmon (fluorescent)	497 (and 522 faint)	576	(<i>Cf.</i> fluorescein).

In all these cases the *neutral* colour is a shade of yellow or brown, not colourless as in the case of the ordinary phthaleins, but none of them when neutral gives a spectrum containing an absorption band; loss of more or less of the violet is all that can be observed. No satisfactory explanation of this *coloured* neutral phase of the sulphonephthaleins has yet been put

forward, but it is probably due to the high ionisation of the $-\text{SO}_3\text{H}$ group as compared with the $-\text{CO}_2\text{H}$ group in the ordinary phthaleins. Thus neutral phenol-sulphonephthalein is probably

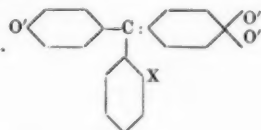


i. e. a substance which is still quinonoid; whereas neutral phenolphthalein is a non-quinonoid lactone, viz.:



and it should be remembered that neutral benzaurine, the parent substance of both, is yellow, like the sulphonephthalein.

It is therefore probable that the deeply coloured *alkaline* solutions of all the phthaleins of every class have the same sort of constitution and one *different* from the above, viz.:



where X is either H, or CO_2' , or SO_3' .

This is the only theory of phthalein colour which explains all cases, including those in which there is only one phenolic ring, such as monoxydiphenylphthalide and phenol-phthalein-monomethyl ether.

As regards the difference in alkaline colour between any phthalein and the corresponding sulphone-phthalein, it will be seen from the table that the change of CO into SO_2 makes little difference, the frequency of the absorption-band being lowered by about $1\frac{1}{2}$ per cent., which is about the same difference as is produced by the entrance of one ortho-methyl group into ordinary phenolphthalein to form phenolcresolphthalein.

OTHER NEW PHTHALEINS AND THEIR SPECTRA.

These were made from 3-nitrophthalic acid (itself made from α -nitronaphthalene).

A table giving the position of their absorption bands is appended :

TABLE III.

No.	Name.	Colour in Alkali.	Wave-length of Absorption Band.	
1	Phenol-3'-nitrophthalein .	Pink	559	Easily bleached by excess alkali.
2	Thymol-3'-nitrophthalein .	Blue	602	Easily bleached by excess alkali.
3	Resorcinol-3'-nitrophthalein	Salmon (green fluorescence)	495	(= 3'-nitrofluorescein).
4	α -naphthol-3'-nitrophthalein	Brown (blue fluorescence)	About 730	—

The 3'-nitro-group has thus quite a small effect, quite different from that of nitro-groups in the phenol rings.

In addition, resorcinol-3'-oxyphthalein (3'-oxyfluorescein) has been made from 3-oxyphthalic acid. Its absorption-band wave-length is λ 489.

Resorcin-saccharein, which is fluorescein with SO_2NH instead of $\text{CO}\cdot\text{O}$, is identical in appearance and spectrum with fluorescein.

NOTE ON THE GENUS *TERFEZIA*; A TRUFFLE FROM THE
KALAHARI.

BY I. B. POLE EVANS, M.A., B.Sc., F.L.S.

(With Plate VII.)

South African truffles belonging to the genus *Terfezia* are well known. Marloth, in his 'Flora of South Africa,' vol. i, p. 26, gives an excellent illustration of one of these—viz. *Terfezia Claveryi*, Chatin, and mentions that—

"Two indigenous species of truffles belonging to the genus *Terfezia* are found in the Kalahari, occurring near shrubs of *Acacia hebeclada*, generally 3-4 inches below the surface of the ground. They are much esteemed as an article of diet when in season (March-June)." In a footnote on this subject Marloth further states: "*Terfezia Claveryi* and *T. Boudieri*, both known from North Africa and used there under the name 'terfaz.'"

As truffles belonging to the first-named species have recently been submitted to me by Dr. Perringuey, Director of the South African Museum, Cape Town, by Dr. Rogers, Director of the Geological Survey, and by G. C. Hunter, Esq., of Dunmurry, Floradale, Griqualand West, a few notes on the genus may not be out of place.

The genus *Terfezia* was created by Tulasne in 1846 as a result of an examination of specimens obtained from Algeria, and which had previously been referred to as *Oogaster algerius*, Corda, and *Tulasneinia leonis*, Zobel.

The genus at present comprises 28 species, and of these at least 11 occur on the African continent. Those recorded from South Africa are: *T. Claveryi*, Chat.; *T. Boudieri*, Chat., from the Kalahari and Windhuk; and *T. Pfeilii*, P. Henn., from Damaraland.

The genus *Terfezia* is readily distinguished from that of *Cheromyces* by possessing broadly ovate or globose asci. In *Cheromyces* the asci are distinctly elongated or lageniform, and arranged in rather simple parallel series. The gleba in *Terfezia* is clearly divided into *massula*, whereas in *Cheromyces* they are not conspicuous.

The plants of *Terfezia Claveryi*, Chat., which I have seen may be described as follows:

Ascomata of variable size and shape, subglobose or more frequently pyriform, shortly and broadly stipitate, 5-7 cm. by 6-9 cm. broad; cortex

yellowish-brown with yellowish cracks; gleba white-yellowish; asci sub-rotund, ovoid, shortly stipitate, 8-spored, $70-87 \times 65-75 \mu$; spores globose, yellowish, episporium very delicately alveolated, $17-23 \mu$ diam.

The plants are usually about the size and shape of an ordinary fig, as is seen in Plate VII, figs. 1 and 2. The average weight is 25-45 gm.

The specimen illustrated by the photograph in Fig. 4 is of exceptional size and more globose in shape; it measured 9 cm. across by 6 cm. deep, and weighed 226 grms. The characteristic shape of the ascus and spores are depicted in Figs. 5 and 6.

Mr. Hunter, in writing to me on the subject, mentioned that the natives found the truffles by observing the cracks in the ground, which, of course, are caused by the growth of these hypogaeal plants.

THE BOTANICAL LABORATORIES OF THE UNION
OF SOUTH AFRICA, PRETORIA.
May 19th, 1917.

EXPLANATION OF PLATE VII.

Illustrating structure of *Terfezia Claveryi*, Chat.

FIG.

- 1 and 2. Typical specimens. Natural size.
3. Specimen cut in half to show character of gleba. Natural size.
4. Large specimen of irregular shape. Slightly reduced.
- 5, a and b. Asci with 8 ascospores. $\times 600$.
- 6, c and d. Ascospores. $\times 1000$.

FIG. 1.

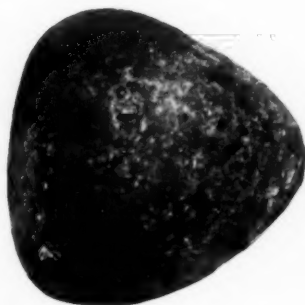


FIG. 2.



FIG. 4.



FIG. 3.

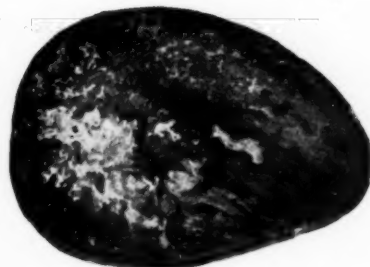


FIG. 5.

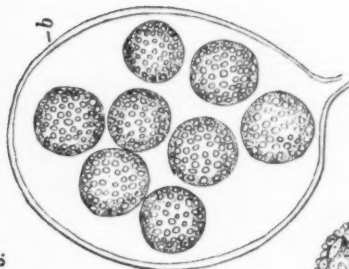
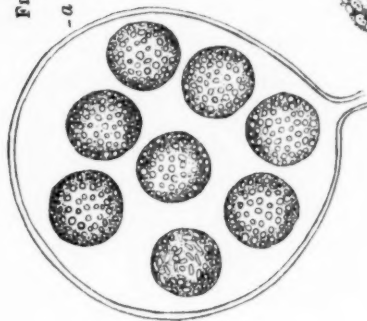


FIG. 6.



J. B. P. Evans del.

NOTE ON THE ABNORMAL DEVELOPMENT OF THE GENITAL
ORGANS OF *JASUS LALANDII* (MILNE EDW.).

By W. VON BONDE, B.A.

(With two Text-figs.)

The peculiar phenomenon here recorded was noticed on a male Crawfish found among the specimens procured for laboratory purposes by the South African College Zoological Department.

An examination of the external features of the animal showed the presence of the usual pair of genital apertures, which in the male are situated on the fifth pair of pereopods or walking legs, but in addition to these two normally developed (*n.g.a.*) openings, there was observed a third abnormally developed aperture (*ab.g.a.*) which occurred on the fourth pereopod on the right side only.

A closer examination of this interesting abnormality revealed the fact that the last or fifth walking leg of the right side, on the basal segment of which was situated one of the normal apertures, took its origin from a point comparatively far in under the sternal elements. The latter thus formed a hard projecting edge over part of the basal segment of the limb, with the result that the genital aperture (*n.g.a.*), which is normally developed close to the articulating point of limb and body, became partially obscured.

The endopodite portions of this particular limb, it may be mentioned, were wanting when the animal was discovered, but there is little doubt that the missing parts had only recently been lost (probably during fishing operations), as the muscle tissues showing at the break were perfectly fresh, presenting a white and soft appearance as contrasted with the hardened, brown surface of the same structures when exposed for any length of time.

The abnormal opening, it may also be mentioned, was developed to perfection as far as the appearance with regard to shape, size, and position is concerned.

A dissection of the specimen was also performed in order to determine the exact relation between this external appearance and the associated internal genital organs. The peculiar development showed the following interesting facts:

The testes of *Jasus lalandii* normally appear as bilaterally, symmetrically disposed organs, each composed of two distinct lobes. The anterior ones, situated laterally to the digestive tract and passing forwards into the head region, are united above the intestine directly behind the mesenteron by a

cross-piece, giving the generative organs the appearance of a slender H. The other pair of lobes lie in the posterior region of the thorax and remain separate throughout their course.

In the case of this abnormal development the testis of the right side was peculiarly constructed. The anterior lobe was stouter and longer than the corresponding one on the left side. The common junction also showed a

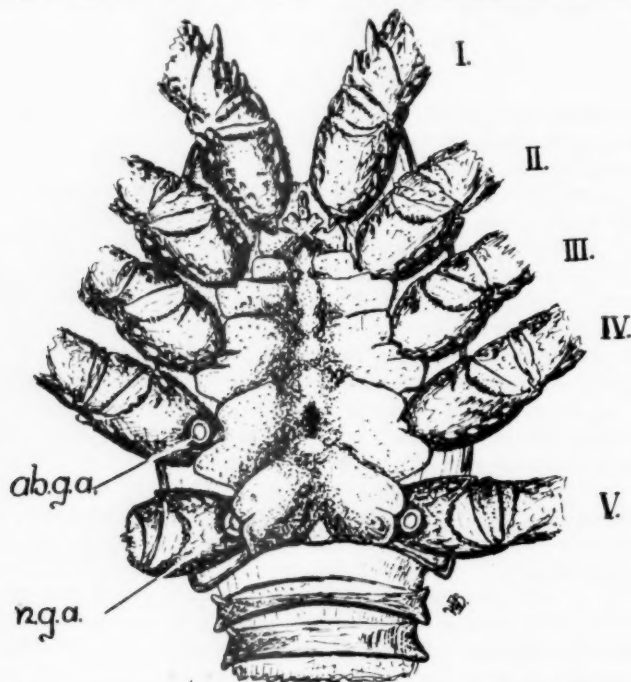


FIG. 1.—Ventral view of *Jasus lalandii* showing normal (n.g.a.) and abnormal (ab.g.a.) genital apertures.

stouter right-hand portion. The posterior lobe, however, presented no marked difference.

The vas deferens of the left side was normally developed, consisting of a much convoluted, fine tube arising from the junction of the testis lobes, and gradually widening until directly above the third pereopod, where the larger coils gave rise to a swollen part, which passed ventralwards to open on the fifth walking leg. A peculiar modification occurred on the right. The first portion of the vas deferens was less convoluted, and hence shorter than the corresponding part of the left. The gradual widening gave rise to a longer,

widened portion of the tube. Directly above the third walking leg this stouter portion bifurcated, sending one thin branch straight to the abnormal

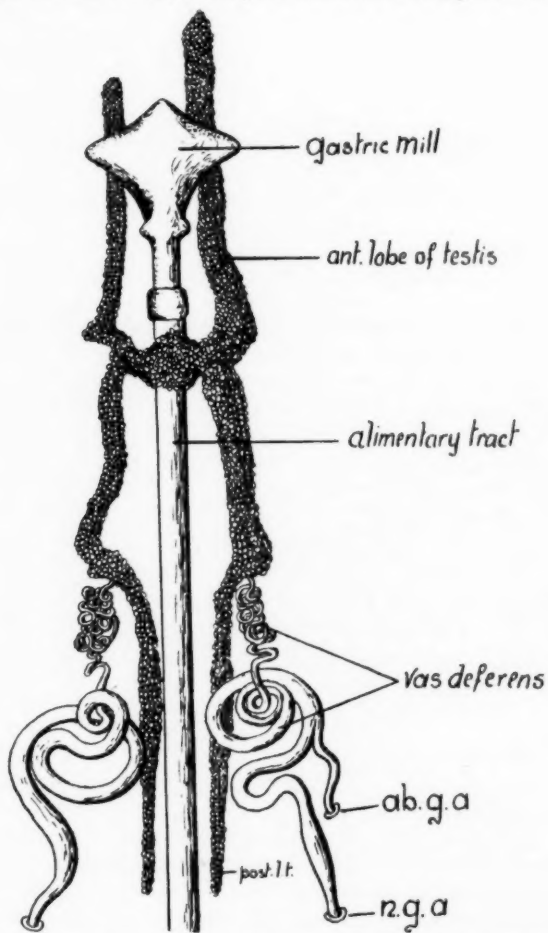


FIG. 2.—Dissected genital system showing relation between genital apertures and vasa deferentia.

aperture of the fourth walking leg, and another, but stouter branch, with an additional coil developed on it, to the normal fifth pereiopod opening. The swollen part noted on the normal vas deferens was developed on this latter part of the genital tube only, and not on the abnormal branch.

COLOUR AND CHEMICAL CONSTITUTION.

PART III.—DERIVATIVES OF THE UNKNOWN ORTHO-PARA-PHENOLPHTHALEIN.

BY JAMES MOIR, M.A., D.Sc., F.I.C.

Ordinary phenolphthalein is a dioxy-derivative of diphenylphthalide, both $-OH$ groups being *para* to the central carbon-atom. Fluorane, on the other hand, is the anhydride of an unstable dioxydiphenylphthalide in which both $-OH$ groups are *ortho* to the central carbon. There should, therefore, exist a third dioxydiphenylphthalide in which one of the $-OH$ groups is *ortho* and the other *para*, to the central carbon, and this, not being capable of forming an anhydride, should be soluble in alkali and should exhibit all the phthalein properties. This paper describes derivatives of this substance, including an attempt to make the substance itself from one of these derivatives. It should be noted that another *ortho-para*-dioxydiphenylphthalide was described in Part I, isomeric with the above but having both $-OH$ groups in the *same* ring, viz. phenylresorcinolphthalein. For convenience of nomenclature, the new substance, in which the *ortho* and *para* $-OH$ groups are in different rings, will be called *o-p*-phenolphthalein.

Preparation of m-methyl-o-p-phenolphthalein (phenolparacresolphthalein).*—Zinc chloride is melted in a round-bottomed flask and distributed in a thin layer by rotation and allowed to solidify. Equal parts of paracresol and *p*-oxybenzoylbenzoic acid are added and the mixture heated in an oil-bath, slowly raising the temperature until the zinc chloride has melted and the whole become a uniform brown gum. On cooling, the zinc chloride is got rid of by digesting with a small quantity of dilute HCl on the steam-bath, cooling and decanting, whereon the organic product is dissolved in dilute caustic soda and reprecipitated with acid, this process eliminating any of the original ingredients which may have escaped reaction. The following are the reactions involved:



* These refer to the central carbon, not to the OH as in Parts I and II.

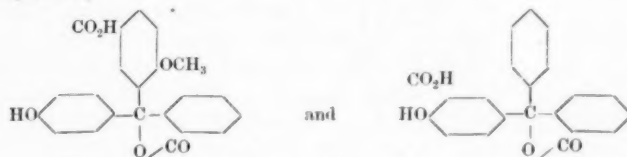
stituent instead of *p*-cresol. The heating must be carefully conducted so as to stop short of the evolution of CO_2 from the carboxylic acid.

The new phthalein gives only a faint colour in ammonia, a stronger base being required to give the highly-coloured phenolic salt in which both the $-\text{CO}_2\text{H}$ and the $-\text{OH}$ groups are combined with alkali. The colour is violet-pink, the absorption-band being at λ 562 in very weak alkali and at λ 570 in strong alkali, and intermediate strengths give intermediate positions. In conc. H_2SO_4 the colour is yellowish-salmon, and the absorption-band at λ 502.

An attempt to make its methyl-ester, starting with methyl *p*-oxybenzoate, apparently gave the same product, the ester being too easily hydrolysed.

The methyl-ether, however, was obtained in small quantity by condensing anisic acid with *p*-oxybenzoylbenzoic acid. It is pink in alkali, bleaching on short standing. The absorption-band is at λ 558 and the one in H_2SO_4 is at λ 503. The close resemblance to oxydiphenylphthalide, of which it is a simple derivative, may be noted.

A still simpler derivative of oxydiphenylphthalide, viz. its carboxylic acid, was made for comparison by condensing benzoylbenzoic acid with salicylic acid. The resulting oxydiphenylphthalide carboxylic acid is colourless in ammonia and pink in caustic alkali (λ 562 in weak and 566 in strong). It is yellow in H_2SO_4 with band at λ 489. Oxydiphenylphthalide itself gives λ 560 in alkali and λ 482 in H_2SO_4 . The formulae of the last two substances are respectively :



The following other derivatives of *o*-*p*-phenolphthalein were made for spectroscopic observation :

- (1) *m*-amino-*o*-*p*-phenolphthalein, from *p*-aminophenol : colour in alkali bottle-green ; broad absorption-band with centre at about λ 575.
- (2) *m*-methylanino-derivative, from "metol" base : colour in alkali as above ; absorption-band covers most of red, but centre is near λ 640.
- (3) ω -carboxylic acid of foregoing, from "glycin" (developer) : colour olive with almost whole of red absorbed.
- (4) *m*-phenyl-derivative, from diphenylol (*p*-phenylphenol) : colour blood-red with green fluorescence ; no definite absorption.
- (5) *m*-nitro-derivative, from *p*-nitrophenol : maroon in alkali with broad band at λ 558.

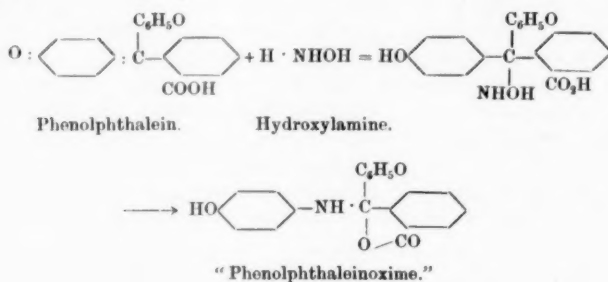
By means of the corresponding *m*-iodo-*o*-*p*-phenolphthalein (made from

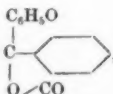
p-iodophenol and *p*-oxybenzoylbenzoic acid) an attempt was made by reduction to obtain the unknown parent substance *o*-*p*-phenolphthalein itself. The product of the condensation was for this purpose digested with a large excess of zinc dust, both acid and alkaline media being tried, and then reoxidised from phthaline to phthalein with H_2O_2 , but I did not succeed in obtaining a product completely free from iodine; and, most remarkable to say, the product resembled ordinary phenolphthalein very closely.

Its alkaline absorption-band is at λ 558 and its H_2SO_4 band is at λ 503, the figures for ordinary phenolphthalein being 554 and 499 respectively. The absorption-band is broader in the supposed *o*-*p*. compound and the colour is comparatively easily bleached by excess of alkali, whereas ordinary phenolphthalein requires strong alkali to bleach it. Further work on this point had to be abandoned on account of the complication introduced by the discovery which I next made, namely that *p*-oxybenzoylbenzoic acid yields a phthalein-like substance when heated alone to over 200°C . or at a lower temperature when heated with a little sulphuric acid. The former observation has already been made by Orndorff and Murray ('J. Amer. Chem. Soc.,' April, 1917, p. 681) in the course of their preparation of another (nearly colourless) anhydride of oxybenzoylbenzoic acid, but these authors, misled by an approximation in colour, state that the second substance is phenolphthalein.

The substance can also be obtained by heating "phenolphthaleinoxime" with a *small* quantity of H_2SO_4 to over 200°C . until a trace withdrawn and dissolved in conc. H_2SO_4 , no longer shows a spectrum absorption at λ 459 but only the band at about λ 500 due to the new substance. Excess of H_2SO_4 must not be used in this preparation, else another substance, which appears to be β -oxyanthraquinone, is produced, which has a very broad absorption-band across F when dissolved in alkali.

It is probable that "phenolphthalein-oxime" is not an oxime at all, but is the *p*-oxyanilide of *p*-oxybenzoylbenzoic acid, its formation being as follows:



This on hydrolysis gives *p*-aminophenol and , the tauto-

meric phthalein-form of *p*-oxybenzoylbenzoic acid (see beginning).

The new phthalein obtained from this acid alone is pinkish-violet in alkali and has a rather broad absorption-band varying between λ 559 and λ 561 as the alkali strength is increased; excess eventually bleaches it. The H_2SO_4 band is at λ 501. It is thus scarcely distinguishable from the supposed *o-p*-phenolphthalein just described, and only differs from ordinary phenolphthalein in a slight degree, having a spectrum similar to that of phenol-*o*-cresolphthalein. The further investigation of this interesting substance must be postponed to another opportunity.

A new property of the phthaleins has been discovered.

It is possible to find a particular considerable strength of alkali in which any phthalein gives a colourless solution in the cold, but which becomes coloured on heating to near the boiling-point and again fades on cooling and allowing to stand. For ordinary phenolphthalein the alkali-strength required is slightly over *N*. strength. Tetriodophenolphthalein requires much weaker alkali to exhibit the phenomenon, and α -naphtholphthalein requires about 2*N* alkali. I cannot suggest any explanation of this, as it is not likely that the hydroxide-ion concentration can be much altered by heat when it is already so high to begin with.

The following is an addition to Part II of this series:

(1) *Phenolphthalein mono-*o*-carboxylic acid* made by condensation of oxybenzoylbenzoic acid with salicylic acid is faintly coloured pink by ammonia (absorption-band λ 561). It is coloured deep violet-pink by caustic alkali (absorption-band λ 567). This substance promises to be a valuable indicator, being like thymolphthalein in properties (*i. e.* suitable for determination of magnesia, etc.), but of a more favourable colour.

(2) The corresponding *dicarboxylic acid* (from phthalic and salicylic acids) described in Part I, has even more marked insensitiveness to alkali, having less affinity than Poirrier-blue. It will be very useful for proving the presence of definite caustic alkalinity at about *N*/100.

(3) *Phenoldiphenylaminephthalein* is also not affected by ammonia; it is violet in soda (absorption-band λ 565 and dulling of centre). It exhibits the same colour in 50 per cent. H_2SO_4 , but is green in conc. H_2SO_4 .

Phenoldimethylanilinephthalein does *not* show the same colour in 50 per cent. H_2SO_4 as in alkali.

Attempts to make mono-ortho-oxy-derivatives of diphenylphthalide by condensing benzoylbenzoic acid with para-substituted phenols gave products which were yellow in alkali and therefore probably not of the expected nature.

SPECTRUM-PHENOMENA IN THE CHROMIUM COMPOUNDS.

BEING PART IV. OF THE SPECTRUM OF THE RUBY AND EMERALD.

BY JAMES MOIR, M.A., D.Sc., F.I.C.

In Parts I to III of my investigation of precious stones published between 1909 and 1912 (Trans. Roy. Soc. S.A., I [2], p. 321, II [3], pp. 271 and 273, and II [4], p. 339), it was shown that the almost unique spectra of ruby and emerald are due to chromium oxide which has been compelled to vibrate in an abnormal or constrained manner, leading to the production of *narrow* absorption bands in the spectrum; the constraining substance in the case of the ruby is crystalline alumina, and in the case of emerald it is beryllium silicate. It was found, in support of this conclusion, that quite a moderate degree of heating abolishes the characteristic spectrum, leaving unaffected the "ordinary" spectrum of chromium. In other words, a hot ruby has much the same spectrum as a solution of chromium sulphate or chloride.

Further investigation has now been made to see if constrained vibration of ordinary chromium oxide could be induced by artificial means, so that the resulting mixture would show narrow absorption bands in the red in addition to the common broad band of unconstrained chromium (between the D and E lines in the case of violet salts and across D in the case of the green salts).

The only previously known cases of these narrow bands are those mentioned in Part II of this work, viz. (1) a hazy band at λ 6960 seen in K_3CrO_3 , and (2) a very hazy band at λ 6800 in chromium borax and "microcosmic" beads and in CrA_3 solution.

The first attempt to make a "constrained" chromium solution was by use of concentrated sulphuric acid. This was successful, the solution showing a red transmission band bounded by two narrow absorption bands just as in the case of the ruby, although the bands are not in the same place. The simplest method of making this solution is to add a few *per cent.* of CrO_3 to concentrated sulphuric acid, heat, and gradually add small quantities of organic matter (*e.g.* starch) until the orange colour has

changed to deep green; it should then be sealed up to prevent absorption of moisture. The spectrum of this solution of $\text{Cr}(\text{HSO}_4)_3$ in H_2SO_4 shows a red transmission band going from λ 6880 to λ 6770, the position of the centre, which is not much affected by varying depth of colour, being at λ 6825. In very strong solutions, however, the two absorption bands bounding this transmission come closer together, and the transmission goes only from λ 6825 to λ 6780. The outer of the two absorption bands is very broad, going from about λ 7350 to λ 6880, and appears rather lighter at λ 7080, as if it really consists of two bands which coalesce in the centre. The inner absorption band is narrow, going from λ 6770 to λ 6660 with greatest intensity at λ 6730. The transmission band of the ruby spectrum is at about λ 6940, *i. e.* considerably further up the spectrum.

A second and even more striking example of the constrained chromium spectrum is obtained when glacial phosphoric acid is used instead of sulphuric acid. The product is a thick green syrup, which solidifies to a "glass" on cooling, and is a solution of $\text{Cr}(\text{PO}_3)_3$ in HPO_3 . Its spectrum shows a transmission band going from λ 6800 to λ 6705, centre λ 6755, very similar to that seen in the ruby, but about twice as broad when seen with an ordinary slit, and also much further down in the red. In this case the whole of the transmission is inside the B line of the solar spectrum, whereas in the H_2SO_4 solution the transmission touches the B line, and the ruby transmission is wholly beyond the B line. The outer absorption band has its centre at λ 6840 and extends from λ 6890 to λ 6800. The inner absorption band crosses the C line of the sun almost symmetrically, its centre being at λ 6590, and its range being from λ 6705 to λ 6480. There is also a third faint band with centre at about λ 6310. This band can also be seen in the emerald. It is probable that by adjusting the concentration of chromium in this medium (HPO_3) and so altering the depth of the absorption bands it would be possible to get a very close imitation of the emerald spectrum. The ruby spectrum, on the other hand, has not been reproduced except by means of alumina (*i. e.* in the artificial ruby); probably another weak acid (*e. g.* boric) would do if it could be induced to dissolve chromium oxide.

None of the other acids tried gave anything characteristic, *viz.* concentrated HCl , HNO_3 , HClO_4 , formic, acetic and citric acids, although a very faint hazy band at λ 6800 was seen in several of these solutions. Most of these comparatively anhydrous solutions have beautiful blue or violet shades, and change to bluish green on boiling.

ON *RANA FUSCIGULA* AND *R. ANGOLENSIS*.

By G. A. BOULENGER, LL.D., D.Sc., F.R.S.

(Published by permission of the Trustees of the British Museum.)

The frogs of the genus *Rana*, taken in the narrowest sense, are represented in South Africa by two closely allied, aquatic species, which may be regarded as the representatives of the Eurasian and North African *R. esculenta*, from which they differ principally in the cranial characters, the nasal bones never meeting on the median line, and a considerable portion of the upper part of the ethmoid remaining uncovered by the fronto-parietal bones, and also in the absence, in males, of external vocal vesicles. These two species have long been known under the names of *R. fuscigula*, D. & B., and *R. angolensis*, Bocage, or *delalandii*, D. & B.,* and until lately they were regarded as perfectly distinct.

Within the last few years, however, doubts have been raised as to their status by Hewitt † and by Andersson, ‡ who independently suggested an intergradation which, if confirmed, would reduce *R. angolensis* to subspecific rank. Mr. J. H. Power has also experienced difficulties in drawing a line between the two species, as they occur near Kimberley, and, at his suggestion, I have examined a portion of the material preserved in the Kimberley Museum, kindly sent to me for study by Miss Wilman, to whom I wish to express my best thanks. §

Adding to this the fine series in the British Museum, I have been able to examine about seventy specimens of *R. fuscigula* and about one hundred of

* I believe two species have been confounded under this name by the authors of the 'Erpétologie Générale,' viz. *R. angolensis* and *R. ozyrhynchus*, as their description of the vomerine teeth applies to the latter and not to the former. However, M. Chabanaud informs me that only two type specimens are now preserved in the Paris Museum, and both agree with the definition of *R. angolensis*. The statement "Cette espèce est fort commune aux environs du Cap de Bonne Espérance" is erroneous, as neither of the two which they appear to have confounded occurs in the Cape Peninsula.

† 'Rec. Albany Mus.,' ii, 1911, p. 206.

‡ 'Svensk. Vet. Ak. Handl.,' xlvii, no. 6, 1911, p. 26.

§ All the specimens had been correctly identified by Mr. Power, but he writes to me that he has received, and actually caught, some which he has been unable to refer to either species, owing, I believe, to his having paid too much attention to the shape of the snout as a diagnostic character.

R. angolensis, and I may say I have had no difficulty in the discrimination of the two species, with the exception of the single specimen, from Natal, described years ago as *R. quacketti*, on the specific distinctness of which I am in doubt. As I can also distinguish the two species in their larval condition, I see no reason for doubting their validity.

The uncertainty to which allusion has been made probably results from the imperfection of previous definitions, and I therefore now propose to give detailed descriptions of the two species, based exclusively on specimens from South Africa and Angola, and also diagnoses by means of which I trust the correct naming of specimens should offer no difficulties. I nevertheless agree with Mr. Hewitt in regarding the two species as the extreme forms of one common stock.

DIAGNOSES OF THE TWO SPECIES.

***Rana fuscigula*.**—Head usually a little broader than long, never longer than broad; interorbital space only a little narrower than the upper eyelid; tibio-tarsal articulation not reaching beyond the tip of the snout; heels not or but slightly overlapping; tibia twice in length from snout to vent, or slightly more or slightly less; toes entirely or nearly entirely webbed, never less than $\frac{3}{4}$ webbed, rarely two phalanges of fourth free; glandular dorsal folds, if present, one only on each side, on the anterior third of the back. Tadpole with 5 or 6 upper and 4 lower series of labial horny teeth.

***Rana angolensis*.**—Head as long as broad or a little longer than broad, very rarely slightly broader than long; interorbital space much narrower than the upper eyelid; tibio-tarsal articulation reaching the tip of the snout or beyond; heels strongly overlapping; tibia $1\frac{1}{2}$ to $1\frac{3}{4}$ times in length from snout to vent; toes $\frac{2}{3}$ to $\frac{3}{4}$ webbed, two or three phalanges of fourth free; back, as far as the sacral region, usually with 6 or 8 more or less prominent, more or less interrupted glandular folds. Tadpole with 3 or 4 upper and 3 lower series of labial horny teeth.

DESCRIPTIONS OF THE TWO SPECIES.

***Rana fuscigula*.**—Vomerine teeth in oval groups or very short transverse or oblique series between or just behind the level of the choanae.

Head usually a little broader than long, never longer than broad, much depressed; snout rounded, rarely obtusely pointed, feebly or scarcely projecting beyond the mouth, as long as the eye or a little longer (up to $1\frac{1}{2}$ times); canthus rostralis obtuse; loreal region very oblique, feebly concave; nostril equidistant from the eye and the tip of the snout, or a little nearer the former; the distance between the nostrils not or but little greater than the interorbital width, which is a little less than that of the upper eyelid;

tympanum very distinct, $\frac{5}{8}$ to nearly once the diameter of the eye and 2 to 4 times its distance from the latter.

Fingers obtusely pointed, first and second equal and as long as or slightly longer than the fourth, third as long as or a little longer than the snout; subarticular tubercles moderate or rather small, moderately prominent. Hind limb moderately long; the tibia-tarsal articulation reaching the eye, the tip of the snout, or between these two points when the limb is stretched forward; the heels meeting or narrowly separated, exceptionally slightly overlapping, when the limbs are folded at right angles to the body; tibia twice in length from snout to vent, or slightly more or slightly less, shorter than the fore limb, as long as the foot or slightly longer or shorter. Toes obtusely pointed, entirely webbed, or at least $\frac{3}{4}$ webbed, in which case one or two phalanges of fourth and one of third and fifth are free; subarticular tubercles small and moderately or feebly prominent; a more or less distinct fold along the inner side of the tarsus; inner metatarsal tubercle elliptical, feebly prominent, $\frac{1}{3}$ to $\frac{1}{2}$ the length of the inner toe; no outer tubercle.

Upper parts with small tubercles and larger warts, some of which often form short, wavy longitudinal folds; sometimes a longer glandular fold on each side of the anterior third of the back, extending to the posterior border of the upper eyelid; a curved fold from the eye to the shoulder; lower parts smooth, thighs granular near the vent.

Brown, greyish-olive, or dark olive above, with dark brown or blackish spots which may form regular longitudinal series on the back and a cross-bar or a chevron between the eyes; a pale green vertebral streak often present; a more or less distinct dark canthal streak and dark and light marblings on the upper lip; a more or less distinct light streak often extends from below the eye to the shoulder; limbs with more or less regular dark cross-bars; hinder side of thighs usually with dark marblings. Lower parts white, spotted, marbled, or vermiculated with blackish; these markings may be restricted to the throat and breast.

Male with internal vocal sacs; the fore limb very strong; a thick pad on the inner side of the first finger, covered during the breeding season with a greyish-brown, velvet-like horny layer, of which a similar but smaller patch is also present on the upper surface of the second finger; small whitish, conical tubercles on the warts on the body.

Tadpole similar to that of *R. esculenta*, but tail often longer, 2 to $2\frac{1}{2}$ times the length of the body, and horny labial teeth in more numerous series, viz. 5 or 6 upper and 4 lower, answering to the following formulae:

$$\begin{array}{cc} 1 & 2 \\ 5-5 & \text{or } 3-3 \\ 1-1 & \text{or } 1-1 \\ 3 & 3 \end{array}$$
 The tadpoles referred to this species by Werner, 'Jen.

Denkschr,' xvi, 1910, p. 295, probably belong to *R. angolensis*.

MEASUREMENTS, IN MILLIMETRES.

British Museum.

Kimberley Museum.

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.
From snout to vent Head	♂	♂	♀	♂	♂	♂	♂	♀	♀	♀	♀	♂	♀	♀	♀	♂	♂	♀	♂	♂	♂	♀	♀	♀	♂	♂	♂	♀
Width of head	61	85	110	73	72	68	49	84	83	79	75	57	70	62	65	94	89	108	66	65	63	95	87	70	61	60	58	68
Snout	21	29	38	27	24	23	18	28	26	28	26	20	24	21	20	31	30	34	21	22	22	31	28	23	21	20	19	22
Eye	22	30	44	28	26	25	19	34	31	34	27	20	26	22	23	34	32	38	22	23	23	36	32	23	22	20	19	25
Inter-orbital width	8	11	15	10	9	9	7	11	10	11	10	7	9	8	9	11	11	13	8	9	8	13	10	9	8	8	7	8
Tympanum	7	9	11	9	8	8	6	10	10	10	9	6	7	7	7	10	9	10	7	8	8	11	10	9	7	7	7	8
Fore limb	3	3	5	3	3	3	2	3.5	3	3.5	3	2.5	3	2.5	3	4	3.5	4	2.5	3	2.5	5	4	4	3	3	2	3
Hind limb	5	7	8	7	6	6	4	7	7	8	6	4	5	4	4.5	8	7	9	5	5	5.5	8	7	6	5	5	5	5
Tibia	36	48	62	46	42	43	30	48	46	47	40	30	39	35	37	51	51	63	37	36	38	50	47	36	34	36	33	39
Foot	83	120	160	119	110	109	78	125	119	123	105	85	103	91	109	149	150	182	98	104	103	126	120	100	99	93	86	106
	28	40	49	37	34	32	23	38	36	38	33	28	33	30	33	47	45	57	32	33	33	42	39	34	32	29	27	34
	30	40	50	40	35	35	25	40	37	39	33	28	34	30	36	48	45	56	32	33	33	39	39	34	32	32	27	35

1. Klipfontein, L. Namaqualand. 2. Cape Town. 3. Cape of Good Hope. 4-11. Deelfontein, Cape Province. 12-14. Schoombie, Middelburg, C.P. 15. Port Elizabeth. 16-18. Vrededorst Road, Orange Free State. 19-25. Schoombie. 26-28. Kalk Bay, Cape Peninsula.

Habitat.—Namaqualand, Cape Province, Orange Free State, Western Transvaal.

A smaller race inhabits Nyassaland and Massailand.

The record of this species from Sierra Leone (Günther, Cat., p. 132; Boulenger, Cat., p. 50) is the result of a misunderstanding. The two specimens in the British Museum labelled as from that part of Africa belong to two distinct species—*R. fuscigula* and *R. occipitalis*. I had overlooked or misread a MS. label on the jar containing these frogs; it bears the following indication, which explains everything: "*Rana fuscigula*, Cape. Ditto, Sierra Leone." The latter locality evidently applies to the *R. occipitalis*, the other specimen being from the Cape of Good Hope.

Rana angolensis.—Vomerine teeth in short or moderately long transverse or oblique series between the choanae, rarely extending a little beyond the level of the posterior borders of the latter.

Head as long as broad or a little longer than broad, very rarely slightly broader than long, rather less depressed than in the preceding species; snout usually obtusely acuminate and projecting beyond the mouth, longer than the eye, rarely not longer; canthus rostralis obtuse; loreal region less oblique than in the preceding, feebly concave; nostril equidistant from the eye and the tip of the snout, or nearer the former; the distance between the nostrils greater than the interorbital width, which is much less than that of the upper eyelid; tympanum very distinct, $\frac{1}{2}$ to $\frac{3}{4}$ the diameter of the eye and $1\frac{1}{2}$ to 3 times its distance from the latter.

Fingers obtusely pointed, first and second equal or, very rarely, first slightly longer than second, third as long as or a little longer or a little shorter than the snout; subarticular tubercles rather small and feebly prominent.

Hind limb variable in length, but usually very long; the tibio-tarsal articulation reaching the tip of the snout or beyond, the heels strongly overlapping; tibia $1\frac{1}{2}$ to $1\frac{7}{8}$ times in length from snout to vent, usually as long as or a little longer than the fore limb, rarely a little shorter, usually as long as or a little longer than the foot. Toes obtusely pointed, about $\frac{2}{3}$ webbed, rarely $\frac{3}{4}$, 2 or 3 phalanges of fourth free; subarticular tubercles small and feebly prominent: a more or less distinct fold along the inner side of the tarsus; inner metatarsal tubercle oval or elliptical, $\frac{1}{3}$ to $\frac{2}{3}$ the length of the inner toe; no outer tubercle.

Upper parts smooth or with small tubercles, usually with more or less prominent, interrupted, glandular, longitudinal folds on the back, 6 or 8 in number, of which a pair beginning from the upper eyelids may be more developed than the others; a curved fold from the eye to the shoulder. Lower parts smooth, thighs granular near the vent.

Coloration very variable. Brown or olive above, usually with dark olive or black spots, which may be large or small, disposed irregularly on the back

or forming regular longitudinal or transverse series; these spots sometimes light-edged; a yellowish or pale green vertebral streak or broad band sometimes present; a dark canthal streak and a dark temporal spot; a light streak from the loreal region to the shoulder usually present, above the upper lip, which is usually brown or blackish spotted with whitish; flanks and hinder side of thighs more or less marbled with blackish, or dark brown, or blackish with white spots; upper surface of limbs with dark cross-bands. Lower parts white, uniform, or with the throat and breast spotted, marbled, or vermiculated with black; in specimens from Angola the throat and breast are usually dark brown or blackish, with the white ground appearing as round spots.

Male with internal vocal sacs, sometimes indicated externally by folds on the sides of the throat; fore limb very strong; a thick pad on the inner side of the first finger, covered during the breeding season with a greyish-brown, velvet-like, horny layer, of which a similar but narrow patch is also present on the upper surface of the second finger; small whitish, conical, or spinose tubercles on the head and body.

The tadpole is distinguished from that of the preceding species by fewer series of horny labial teeth, viz. 3 or 4 upper and 3 lower, answering to the

following formulae: $\begin{smallmatrix} 1 \\ 3-3 \\ 1-1 \\ 2 \end{smallmatrix}$, or $\begin{smallmatrix} 1 \\ 2-2 \\ 1-1 \\ 2 \end{smallmatrix}$, or $\begin{smallmatrix} 1 \\ 2-2 \\ 3 \end{smallmatrix}$. The tail is often largely blotched

with blackish.

Habitat.—Eastern parts of the Cape Province, Natal, Zululand, Transvaal, Bechuanaland, Rhodesia, Angola, Portuguese East Africa, Nyassaland.

Mr. Power informs me that *R. angolensis* is plentiful at the Vaal River, 16 miles north of Kimberley, and at the Modder River, 24 miles south of Kimberley, and that *R. fuscigula* occupies the intermediate area. He has however, taken specimens of both species in the same pool at the junction of the Modder and Riet Rivers, and there are other parts of the country where their range overlaps.

Rana theileri, Macquard, 'Bull. Mus. Paris,' 1906, p. 252, from the Transvaal, is certainly a synonym of *R. angolensis*.

I am now inclined to regard, provisionally, *R. quекetti*, Bouleng., 'Proc. Zool. Soc.,' 1894, p. 643, Pl. XXXIX, fig. 1, founded on a single specimen from Pietermaritzburg, as an abnormal *R. angolensis*. It agrees with this species in the shape of the head, in the very narrow interorbital space, and in the toes, which are only about $\frac{2}{3}$ webbed. But the hind limb is shorter, as in *R. fuscigula*, the tibio-tarsal articulation reaching between the eye and the nostril, and the tibia measuring a little less than $\frac{1}{2}$ the length from snout to vent.

MEASUREMENTS, IN MILLIMETRES.

	British Museum.															Kimberley Museum.																															
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.	25.	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.	37.	38.	39.	40.	41.	42.					
From snout to vent	♂	♂	♀	♀	♀	♀	♀	♀	♀	♀	♀	♂	♂	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♂	♂	♂	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀	♀		
Head	53	46	73	59	61	64	64	77	55	64	53	53	51	77	77	71	60	53	68	45	80	88	52	63	75	69	67	75	65	92	89	46	58	40	45	63	62	49	48	73	71	72					
Head	19	17	25	19	21	23	22	26	21	24	20	18	24	25	24	24	19	22	16	26	31	18	25	26	23	22	25	22	29	27	17	21	19	17	23	23	18	18	26	25	24						
Width of head	19	16	25	19	20	21	22	26	18	24	18	18	18	24	23	23	21	19	22	16	26	32	17	24	29	20	20	23	22	27	26	16	20	19	17	22	22	17	17	24	23	24					
Snout	7.5	7	11	8	8	10	9	12	9	8	8	7	11	10	9	9	7.5	9	7	10	14	7.6	11	11	9	9	11	10	13	12	6	8	7.6	5	10	9	7.5	10	9	7.5	10	10	10				
Eye	7	6	8	6	8	8	8	7	7	7	6	8	8	7.6	8	8	7.6	8	6	11	6	8	8	8	8	8	10	9	6	7	7.0	5	8	8	7	7	7	7	7	7	7	7	7	7	7	7	
Interorbital width	2	2.25	2	2.25	2	2.25	2	3.25	2	2.25	2.25	2.25	3.25	3	2	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	2.25	
Tympanum	4	4	6	4	5	5	5	6.5	5	4	4.5	5	5	5	5	5	4	5	4	7	4	6	5	6	5	5	5.6	6.5	4	5	4	5	4	5	4	5	4	5	4	4	6	6					
Fore limb	35	30	42	33	34	35	35	47	34	36	37	36	31	41	37	40	31	35	28	26	44	52	29	45	44	42	40	40	39	45	44	25	33	32	28	35	35	28	27	39	39	39					
Hind limb	105	82	127	100	105	108	108	152	110	105	108	107	90	135	129	137	101	103	127	88	133	160	90	130	140	123	122	125	117	158	139	79	104	94	85	115	117	93	87	123	128	129					
Tibia	33	27	42	32	35	37	35	51	37	35	35	36	29	45	42	44	32	33	42	28	46	54	29	40	43	39	38	40	37	51	46	25	34	30	26	38	38	39	29	43	44	40					
Foot	30	27	39	32	34	35	34	45	34	35	34	34	30	42	40	41	30	33	26	27	41	59	30	40	45	40	38	40	37	51	44	24	34	30	26	35	37	29	28	40	40	37					

1-4. Port Elizabeth. 5. Riverton, near Kimberley. 6-7. Watersmeet, Vryburg, Brit. Bechuanaland. 8. Durban. 9. Pietermaritzburg.
 10. Ross's Kloof, Natal. 11-16. Sibiutani, Zululand. 17. Pretoria. 18-20. Walkersdorp, Transvaal. 21. Turloep, Transvaal.
 22. Mafusi, Gazaland. 23. Guanyaya R. N. of Bulawayo. 24-25. Angola. 26-27. Caconda, Angola. 28-31. Pungo Andongo, Angola. 32. Riverton, near Kimberley. 33-36. Modder R., Kimberley. 37. Watersmeet, Vryburg, Brit. Bechuanaland. 38-39. Taung, Brit. Bechuanaland. 40-41. Pochetsdorp, Transvaal. 42. Marandellas, S. Rhodesia.

HENRY HAROLD WELCH PEARSON, F.R.S.

A familiar figure with an almost boyish gait no longer tramps the Avenue, no longer climbs the winding stair, no longer tends The Gardens.

Henry Harold Welch Pearson lies at rest on the Cycad hill at Kirstenbosch.

It is hard, indeed, to contemplate this untimely loss, not only to South Africa, but to the botanical world in general, for the name of Pearson will for all time be associated with the great Cycad family, the Gnetales, and South Africa's National Gardens.

Pearson came to South Africa, a comparatively young man, some fourteen years ago; he has thus laid down a strenuous and fruitful life with an all too sudden suddenness.

Born at Long Sutton, Lincolnshire, in 1870, he died in Cape Town after a short illness on November 3, 1916. He was educated privately and matriculated in the University of London in 1889.

Entering the University of Cambridge as a non-collegiate student in October, 1893, he secured a First Class in Part I of the Natural Science Tripos, and entered Christ's College in 1896. In Part II of the Natural Science Tripos he again obtained a First Class, and was elected a Foundation Scholar and Darwin Prizeman at his college. Almost immediately after this he proceeded to Ceylon as a Wort's Travelling Scholar of the University, and spent some six months there chiefly engaged on oecological studies. For his work in Ceylon he was awarded the Walsingham medal. He returned to Cambridge in 1898, and was appointed Assistant Curator of the University Herbarium under Marshall Ward, for whom he always bore the greatest admiration and respect. Very soon after this he was elected Frank Smart Student in Botany at Gonville and Caius College.

In 1899 he was appointed Assistant for India in the Herbarium at Kew.

In 1903 he came to South Africa to fill the newly-founded Harry Bolus Chair of Botany at the South African College, Cape Town, and ten years later he combined this post with that of Honorary Director of the National Botanic Gardens at Kirstenbosch.

The field of investigation covered by Pearson includes work of first-class importance dealing with histology, physiology, oecology, and geographical distribution; in fact, it was characteristic of him that he always attempted to look at the problem he had in hand from all points of view.

His first published paper dealt with the anatomy of the seedling of

Bowenia, a remarkable Australian Cycad, and even at that time it is clear that many problems relating to the physiology and oecology of the Cycads must have been prominent in his mind. This was Pearson's first piece of research work at Cambridge and it is interesting to note that he never forsook this group, which had for him a peculiar fascination.

His account of the botany of the Ceylon Patanas is of exceptional interest to the South African botanist in that the flora of the Patanas bears in many respects a striking resemblance to that of the High Veld in South Africa, and one regrets that other duties and strenuous work prevented Pearson from pursuing the oecological studies begun in Ceylon further with respect to the grassland of this country, especially as the same problems and questions that presented themselves in Ceylon must have occurred again and again to him here.

His study of the Patana vegetation aroused an interest in Xerophytes, which he retained to the end. In particular, he drew attention to the absence of plants with bulbs or tubers from the flora of the dry Patanas, and in this respect it must have formed a striking contrast to that which he encountered in his travels in South Africa.

While at Kew Pearson was engaged chiefly in systematic botany, but he also undertook a morphological study of the double pitchers found in certain members of the genus *Dischidia*, and from his examination of dried and scanty material he put forward the novel theory that the double pitchers represented Xerophytic characters. His systematic work included the working up of the Verbenaceae for the 'Flora Capensis.'

Coming to South Africa in 1903, Pearson's first botanical publications dealt with the Verbenaceae, and very soon afterwards he published an instructive and useful paper on the South African Cycads.

At an early date he turned his attention to the study of the flora of South-West Africa, where he travelled and explored the desert regions inhabited by *Welwitschia* and other no less remarkable Xerophytes. His observations on *Welwitschia* were not confined to the field, but were supplemented by a series of classical papers detailing the results of anatomical and histological studies. He first made himself thoroughly familiar with the natural history of the plant, journeying to Damaraland on no less than three occasions solely with the object of studying it in the field.

As a result of these observations he concluded that full-grown plants reached an age of considerably over a hundred years, that they were pollinated by insects, and that when growing in contact they frequently formed natural grafts. The material which was acquired on these different expeditions formed the basis for a most exhaustive study in his laboratory at Cape Town of the details connected with the process of fertilisation in *Welwitschia*, and conclusions of extraordinary interest relating to the organisation and structure of the endosperm were put forward by Pearson.

His paper on the subject communicated to the Royal Society of London may well be regarded as his masterpiece. It is here that he cleverly disposed of a puzzling and troublesome structure by his term "trophophyte."

In like manner in his first paper, also communicated to the Royal Society, he was responsible for introducing the term "prothallial-tube."

At this time, when he was so actively engaged in the histological study of the endosperm of *Welwitschia*, he had naturally taken advantage of the enthusiasm which he had instilled into his more promising students to prosecute research of a similar nature amongst other South African genera, and during this time a series of papers dealing with the development of the embryo-sac appeared.

Pearson's observations on *Welwitschia* at once raised many points with regard to its alliance with *Gnetum* and their affinity to the Angiosperms. Both these phases he entered into fully, and even predicted that *Gnetum* would show similar fertilisation phenomena as pertained in *Welwitschia*.

Having disposed of the botanical details of the fertilisation of *Welwitschia*, Pearson with characteristic thoroughness turned his attention to *Gnetum*. Here again the first step that he undertook was a study of the life-history of *Gnetum africanum*. With this object in view he visited Angola in 1909, and obtained material for such an investigation as he had previously carried out on *Welwitschia*.

From his examination of *Gnetum africanum* and two other species he concluded that *Gnetum* showed a much closer degree of affinity with *Welwitschia* than with *Ephedra*. His first paper left many interesting points in connection with the male gametophyte still to be cleared up. By this time *Gnetum* was being examined by several workers, and before Pearson's second paper on *Gnetum* dealing with the morphology of the inflorescence and flower appeared, Thompson in America published the results of his examination of the embryo-sac and male gametophyte, from which it was evident that the conditions in *Gnetum* resembled those in *Welwitschia*, as Pearson had many years before predicted.

Pearson, however, had not been slow in unravelling the puzzling and difficult histological details connected with the reproduction of *Gnetum*, for, in his paper presented to the Linnean Society on June 4, 1914, he showed very clearly that "the primary endosperm of *Gnetum* is in all respects homologous with the primary endosperm of *Welwitschia*," and his suggestions that such fertilisation phenomena might be associated with the polar nuclei in Angiosperms have given rise to much reasonable discussion and speculation on these points.

Two further papers on *Gnetum* were written by Pearson prior to his death, and are as yet still in the press.

Pearson, in his travels in South-West Africa, did not confine himself to the mere collection of material for further study in the laboratory or com-

parison in the herbarium, but he set himself to inquire in the field into certain definite physiological phenomena connected with the internal temperatures of plants growing under desert conditions. Observations made in December on *Euphorbia virosa* and *Aloe dichotoma* revealed the fact that the internal temperature of the former rose as high as 51.5°C . *Euphorbia* also showed an excess of internal temperature over that of shade by 15.35°C ., while *Aloe* did not vary more than 5°C . He further made the interesting observation that when wounded a considerable drop in internal temperature took place in *Euphorbia*; this, Pearson explained, was partly due to evaporation at the wound and partly to the expansion of gas in the air chambers.

In addition to his more purely scientific work Pearson also undertook an investigation of the "Witchweed," a phanerogamic parasite which was causing damage to maize crops in the Transvaal. The botanical characters of the parasite were soon worked out with a skill and clearness of vision that were characteristic of the man, but through no fault of his no practical solution to the problem was arrived at.

As a lecturer Pearson had that rare gift of ready and lucid exposition. He thought quickly and clearly. He took an active though unobtrusive part in the affairs of the Cape of Good Hope University, and latterly was much absorbed in the selection of the site of the University of Cape Town on the Groot Schuur Estate and its future relationship to the National Botanic Gardens.

To the South African public Pearson was best known as the Director of the National Gardens at Kirstenbosch, which office he filled in an honorary capacity, and nothing pleased him more when opportunity arose than to show both local and distant visitors the work that was then in hand at the Gardens and discuss plans and possibilities for the future, always both humorous and courteous to all alike.

By Pearson's untimely death the Royal Societies of both London and South Africa have been robbed of a Fellow whose name will always rank with those of Thunberg, Burchell, Ecklon, Zeyher, Drège, and Bolus, whether he be regarded as a naturalist, botanist, traveller, explorer, or teacher.

He is lamented by all who knew him, and it is difficult to contemplate Cape Town and the National Gardens without his cheery and enlivening presence. Ill could the country afford to lose him. I. B. P. E.

LIST OF PAPERS.

1898. "Anatomy of the Seedling of *Bowenia spectabilis*," Hook, f. ('Ann. Bot.', xii, pp. 475-490, tt. 27-28).
1898. "Apogeotropic Roots of *Bowenia spectabilis*," Hook, f. ('Rep. Brit. Assoc. Adv. Sci.', p. 1066).

- 1899 & 1903. "Botany of the Ceylon Patanas," part 1 ('Journ. Linn. Soc.,' vol. xxxiv., pp. 300-365, with map); part 2, by J. Parkin and H. H. W. P. (*loc. cit.*, vol. xxxv, pp. 430-463, tt. 11-12).
1900. "S. Hedin's Reisen in Zentralasien: die botanischen Ergebnisse," bearbeitet von W. Botting Hemsley and H. H. W. P. ('Petermann's Geogr. Mitteil. Ergänzungsab.,' xxviii, pp. 372-375).
1901. "Description of *Clerodendron Curtisii*" ('Kew Bulletin,' p. 142).
1901. "On a Small Collection of Dried Plants obtained by Sir Martin Conway in the Bolivian Andes," by W. Botting Hemsley and H. H. W. P. ('Journ. Linn. Soc.,' vol. xxxv, pp. 78-90, with map).
1901. "The Flora of Tibet or High Asia, being a Consolidated Account of the various Tibetan Botanical Collections in the Herbarium of the Royal Gardens, Kew, together with an Exposition of what is known of the Flora of Tibet," by W. Botting Hemsley, assisted by H. H. W. P. ('Journ. Linn. Soc.,' vol. xxxv, pp. 124-265, with map).
- 1900 & 1901. "Descriptions of *Cochlearia Hobsoni*, *Geophila pilosa*, *Cuscuta Hygrophilae*, *Pentaphragma albiflorum*, and *Vitex mooiensis*" (Hook., 'Ic. Plant.,' vol. xxvii, tt. 2643, 2691; vol. xxviii, tt. 2704-2706).
- 1901 & 1910. "Verbenaceae (of South Africa)" (Dyer, 'Flora Capensis,' vol. v, sect. 1, pp. 180-226).
1902. "On some Species of *Dischidia* with double Pitchers" ('Journ. Linn. Soc.,' vol. xxxv, pp. 375-390, t. 9).
1903. "The Teaching of Botany" ('Rep. S. African Assoc. Adv. Sci.,' pp. 312-316).
1903. "The Double Pitchers of *Dischidia Shelfordii*" ('Ann. Bot.,' vol. xvii, pp. 617-618).
1905. "South African Verbenaceae" ('Trans. S. African Phil. Soc.,' vol. xv, pp. 175-182).
- 1905 & 1906. "Notes on some South African Cycads" ('Rep. S. African Assoc. Adv. Sci.,' p. 260; 'Trans. S. African Phil. Soc.,' vol. xvi, pp. 341-354, tt. 6-8 and 1 text-fig.).
1906. "Some South African Cycads: their habitats, habits, and associates" ('Rep. Brit. Assoc. Adv. Sci.,' pp. 738-739).
1906. "Some Observations on *Welwitschia mirabilis*," Hooker f. ('Phil. Trans. R. Soc. B.,' vol. cxviii, pp. 265-304, tt. 18-22).
1906. "Descriptions of *Connaropsis acuminata*, *Cuscuta Upcraftii*, *Euthemis ciliata*, *Semecarpus cinerea*, and *Swintonia puberula*" ('Kew Bulletin,' pp. 2-5).
1907. "The Living *Welwitschia*" ('Nature,' vol. lxxv, pp. 536-537, with 3 figs.).
1907. "A Botanical Excursion in the Welwitschia Desert" ('Rep. Brit. Assoc. Adv. Sci.,' p. 685).

1907. "Some Observations on the Welwitschia Desert" (abstract) ('Rep. S. African Assoc. Adv. Sci.,' p. 116).
1907. "Research on South African Cycads and on *Welwitschia*": Report to the Committee ('Rep. Brit. Assoc. Adv. Sci.,' pp. 408-409).
1908. "A Note on the Morphology of Endosperm" ('Rep. Brit. Assoc. Adv. Sci.,' p. 914).
- 1908 & 1909. "Further Observations on *Welwitschia*" ('Proc. R. Soc. Lond. B.,' vol. xxx, pp. 530-531; 'Phil. Trans. R. Soc. Lond. B.,' vol. cc, pp. 331-402, tt. 22-30). (Abstract.)
1909. "A Botanical Journey in South-West Africa" ('Gard. Chron.,' vol. lvi, pp. 369-370, 401-402, 414-415, figs. 160, 160 a, 176-178, 182-184).
1909. "Percy Sladen Memorial Expedition in South-West Africa, 1908-9" ('Nature,' vol. lxxxi, pp. 466-467, 499-500, with 4 figs. and 2 sketch-maps).
1910. "The Travels of a Botanist in South-West Africa: Percy Sladen Memorial Expedition, 1908-9" ('Geogr. Journ.,' vol. xxxv, pp. 481-511, with 12 figs.).
1910. "*Welwitschia mirabilis*" ('Gard. Chron.,' xlvii, pp. 49-51, figs. 31-33 and Suppl. illust.).
1910. "A National Botanic Garden (for South Africa)": Presidential Address ('Rep. S. African Assoc. Adv. Sci.,' pp. 37-54; abstract in 'Kew Bulletin,' pp. 372-380).
1910. "The Embryo of *Welwitschia*" ('Ann. Bot.,' vol. xxiv, pp. 759-766, t. 64 and 2 text-figs.).
1910. "Thymelaeaceae (of Tropical Africa)" (Dyer, 'Flora of Tropical Africa,' vol. vi, sect. 1, pp. 212-255).
1911. "Preliminary Report on an Investigation of the Life-History of the Rooibloem or Witchweed (*Striga lutea*, Lour.)" ('Agric. Journ. Union S. Africa,' vol. ii, pp. 266-268).
1911. "A State Botanic Garden" ('The State of South Africa.')
1911. "Harry Bolus, D.Sc., F.L.S. (Obituary Notice, with a list of his botanical journeys, by L. Kensit, and a bibliography)" ('Rep. S. African Adv. Sci.,' pp. 67-79, with portrait).
1911. "Through Little Namaqualand with the Vasculum and the Camera" ('Gard. Chron.,' vol. l, pp. 61, 62, 124-125, 166-167, 190-191, 200-201, figs. 30-33, 57-61, 79-81, 89-91, 97-99).
1911. "On the Collections of Dried Plants obtained in South-West Africa by the Percy Sladen Memorial Expeditions, 1908-1911" ('Ann. S. African Mus.,' vol. ix, pp. 1-19, with map).
1911. "Itinerary of the Percy Sladen Memorial Expedition to the Orange River, 1910-1911" ('Ann. S. African Mus.,' vol. ix, 1911, pp. 21-90, tt. 1-2).

1912. "La Vaillant's Grotto at Heerenlogement" ('Geogr. Journ.,' vol. xxxix, pp. 40-47, with sketch-map and 3 figs.).
1912. "On the Microsporangium and Microspore of *Gnetum*, with some Notes on the Structure of the Inflorescence" ('Ann. Bot.,' vol. xxvi, pp. 603-620, t. 60, and 6 text-figs.).
- 1912 & 1913. "List of Plants collected in the Percy Sladen Memorial Expeditions, 1908-9, 1910-11 (Portulacaceæ), by H. H. W. P. and E. L. Stephens" ('Ann. S. African Mus.,' vol. ix, pp. 30-35); Verbenaceæ (*loc. cit.*, vol. ix, pp. 183-184).
1912. "On the Rooibloem (Isona or Witchweed)" ('Agric. Journ. Union S. Africa,' vol. iii, pp. 651-655; 'Union S. Africa, Dep. Agric.' (leaflet), no. 30, pp. 1-7).
1913. "The Problem of the Witchweed" ('Agric. Journ. S. Africa,' vol. vi, pp. 803-805; 'Union S. Africa, Dep. Agric.' (leaflet), no. 40, (pp. 1-34, with 9 figs.).
1913. "Note on the Localities visited by the Percy Sladen Memorial Expedition to the Khamiesberg, Giftberg, and Oliphant's River Mountains, September, 1911" ('Ann. S. African Mus.,' vol. ix, pp. 129-131).
1913. "The National Botanic Garden (for South Africa)" ('Gard. Chron.,' vol. liv., pp. 150-151, with sketch map).
1914. "On the Flora of the Great Karasberg": Introduction ('Ann. Bolus Herb.,' vol. i, pp. 1-8).
1914. "Observations on the Internal Temperatures of *Euphorbia virosa* and *Aloe dichotoma*" ('Ann. Bolus. Herb.,' vol. i, pp. 41-66).
1915. "Note on the Inflorescence and Flower of *Gnetum*" ('Ann. Bolus Herb.,' vol. i, pp. 152-172, tt. 24-26).
1915. "Communication on the Experimental Use of Plants for Fixing Moving Sand" ('Min. Pro. of the S. African Soc. Civil Engineers,' pp. 268-270).
1915. "Notes on the Morphology of Certain Structures concerned in Reproduction in the Genus *Gnetum*" ('Journ. Linn. Soc.,' vol. xliii, pp. 55-56).

WORKS.

1914-16. Editor of:

- (1) 'Annals of the Bolus Herbarium,' vols. i-ii, part 1.
- (2) 'Journal of the Botanical Society of South Africa.'

THE TORSION PROBLEM FOR BODIES OF REVOLUTION.

BY E. T. STEGMANN.

(Thesis approved for the Degree of Doctor of Science in the University of the Cape of Good Hope.)

(With Plates A, B, VIII-XVIII.)

INTRODUCTION.

The problem which we are to consider deals with the distribution of stresses and strains in an isotropic body of revolution of infinite length under the action of terminal couples and free from body forces.

As far as I am aware, an analytical solution of the problem has been given only in the case of a circular cylinder, the solution following from Saint-Venant's general theory of the torsion of cylinders.*

The general theory of the torsion of such bodies has been worked out by Föppl† and Williers.‡ They have shown that in such a body, where the stresses and strains are obviously independent of θ , we have the equation,

$$\frac{\partial}{\partial z} \left(\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial r} \left(\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial r} \right) = 0,$$

where

$$\begin{aligned} r\theta &= - \frac{1}{r^3} \cdot \frac{\partial \psi}{\partial z} \\ \theta z &= + \frac{1}{r^3} \cdot \frac{\partial \psi}{\partial r}, \end{aligned}$$

and further that

$$\widehat{rr} = \widehat{\theta\theta} = \widehat{zz} = \widehat{rz} = 0,$$

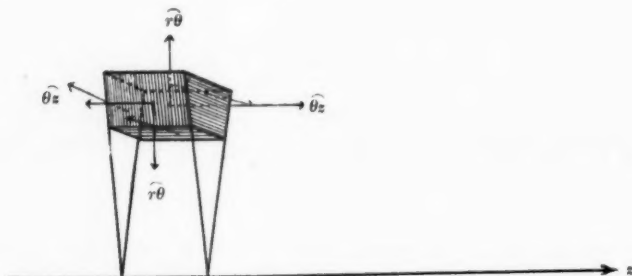
where \widehat{rr} , . . . , \widehat{rz} . . . are the stress-components referred to cylindrical co-ordinates r , θ , z .

* Love, "Theory of Elasticity," § 86, α ; § 221, α . See also Webster, "Dynamics," § 184, β .

† Föppl, 'Münch. Ber.' 35 (1905), p. 249. 'Zeitsch. der Ver. der Ing.' 50 (1906), p. 1032.

‡ Williers, "Diss.," Göttingen (1908). 'L. f. Math. u. Physik,' 55 (1907), p. 225.

The stress-components acting upon an elementary cylindrical element will therefore be as shown in the figure.



The equations may immediately be deduced as follows :

Since the isotropic body is in equilibrium, and since no body forces are acting, the equation of equilibrium * that contains $\widehat{r\theta}$ and $\widehat{\theta z}$ reduces to

$$\frac{\partial \widehat{r\theta}}{\partial r} = \frac{\partial \widehat{\theta z}}{\partial z} + 2 \cdot \frac{\widehat{r\theta}}{r} = 0,$$

$$i. e. \quad \frac{\partial}{\partial r} (r^2 \cdot \widehat{r\theta}) + \frac{\partial}{\partial z} (r^2 \cdot \widehat{\theta z}) = 0.$$

A function ψ therefore exists such that

$$r^2 \cdot \widehat{r\theta} = - \frac{\partial \psi}{\partial z} \text{ and } r^2 \cdot \widehat{\theta z} = + \frac{\partial \psi}{\partial r},$$

$$i. e. \quad \left. \begin{aligned} \widehat{r\theta} &= - \frac{1}{r^2} \cdot \frac{\partial \psi}{\partial z} \\ \widehat{\theta z} &= + \frac{1}{r^2} \cdot \frac{\partial \psi}{\partial r} \end{aligned} \right\} \quad \quad \quad (1)$$

At any point the axis of the resultant of the $\widehat{r\theta}$, $\widehat{\theta z}$ stresses is normal to lines given by

$$\frac{dz}{\partial z} = \frac{dr}{r\theta},$$

which lines are therefore lines of zero stress as far as $\widehat{\theta z}$ and $\widehat{r\theta}$ are concerned, and which will be completely free from stress if the other equations

$$\widehat{rr} = \widehat{\theta\theta} = \widehat{zz} = \widehat{rz} = 0$$

also hold.

The theory, however, given below is independent of these last equations.

Hence we have for the stress lines (lines of zero stress) dependent on $\widehat{r\theta}$ and $\widehat{\theta z}$,

* Love, "Theory of Elasticity," § 59.

$$\frac{dz}{\frac{1}{r^2} \cdot \frac{\partial \psi}{\partial r}} = \frac{dr}{\frac{1}{r^2} \cdot \frac{\partial \psi}{\partial z}}.$$

i. e. $\frac{\partial \psi}{\partial z} \cdot dz + \frac{\partial \psi}{\partial r} \cdot dr = 0.$

i. e. $\downarrow = \text{constant}.$

Again, in an isotropic solid the strain-energy function is an invariant for all transformations from one set of orthogonal axes to another. Hence we conclude that if W is the strain-energy function,

$2W = (\lambda + 2\mu)(e_{rr} + e_{\theta\theta} + e_{zz})^2 + \mu(e_{r\theta}^2 + \dots - 4e_{r\theta} \cdot e_{\theta z} \dots),$
where $e_{rr} \dots, e_{r\theta} \dots$, are the components of strain, and λ and μ the elastic constants.*

Further,

$$\begin{aligned} \widehat{rr} &= \frac{\partial W}{\partial e_{rr}}, & \dots & \dots & \dots & \dots \\ \widehat{r\theta} &= \frac{\partial W}{\partial e_{r\theta}}, & \dots & \dots & \dots & \dots \end{aligned}$$

We therefore immediately get

$$\begin{aligned} \widehat{rr} &= \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{rr} \\ &\dots \dots \dots \dots \dots \dots \\ \widehat{r\theta} &= \mu e_{r\theta} \\ &\dots \dots \dots \dots \dots \dots \end{aligned}$$

For the strain-components $e_{r\theta}$ and $e_{\theta z}$ we therefore have

$$e_{r\theta} = \frac{1}{\mu} \cdot \widehat{r\theta}, \quad e_{\theta z} = \frac{1}{\mu} \cdot \widehat{\theta z}.$$

If u_r, u_θ, u_z are the components of displacement, then for u_θ we have the equations,

$$\begin{aligned} e_{r\theta} &= \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} = \frac{1}{\mu} \cdot \widehat{r\theta} \\ e_{\theta z} &= \frac{\partial u_\theta}{\partial z} = \frac{1}{\mu} \cdot \widehat{\theta z} \end{aligned} \quad (2)$$

Eliminating u_θ from (2) we get,

$$\frac{\partial}{\partial z} \left(\frac{\widehat{r\theta}}{r} \right) - \frac{\partial}{\partial r} \left(\frac{\widehat{\theta z}}{r} \right) = 0.$$

Making use of 1 we get,

$$\frac{\partial}{\partial z} \left(\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial r} \left(\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial r} \right) = 0 \quad (3)$$

* Love, "Theory of Elasticity," § 59.

Equations (1), (2), and (3) will still hold, even if \widehat{rr} , $\widehat{\theta\theta}$, \widehat{zz} , and \widehat{rz} do not vanish, provided only that the stresses, strains, etc., are independent of θ .

If, in addition, we put

$$\widehat{rr} = \widehat{\theta\theta} = \widehat{zz} = \widehat{rz} = 0,$$

it immediately follows from the equations of compatibility that

$$u_r = u_z = 0.$$

Williers integrated 3 graphically in some special cases.

The rest of this paper is concerned with the analytical and graphical determination of the stress-lines, stress-components, and displacement in certain selected cases.

SECTION 1.

Since we assume that the surface of revolution is free from external forces, the stress-lines cannot cut the surface anywhere. That is, in a meridian plane, the boundary lines of the body of revolution will themselves be stress-lines. The stress-lines are, however, given by $\psi = \text{constant}$, and hence, if we can find any particular solution ψ_1 of equation (3), then any of the family of lines $\psi_1 = \text{constant}$ may be taken as the boundary lines of the body of revolution in a meridian plane, provided, however, that ψ_1 has no singularities within the body. In such a body the stresses and the displacement will be known, these being given by equations 1 and 2 respectively.

In all the cases considered, the body assumes at its two distant ends the shape of circular cylinders, and the distribution of the terminal couples over the end faces is determined in each case.

As Saint-Venant has shown, however, the problem in each case is independent of the distribution of the terminal couples, except at points near the distant ends, so that the results hold at all points sufficiently far from the ends to avoid the end effects, whatever be the terminal distribution.

SECTION 2.

Five-dimensional Hydrodynamical Analogy.

If we consider equation 3, the similarity between it and the equation for the Stokes' stream function,

$$i. e. \quad \frac{\partial}{\partial z} \left(\frac{1}{r} \cdot \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial r} \left(\frac{1}{r} \cdot \frac{\partial \psi}{\partial r} \right) = 0,$$

immediately strikes us, the only difference being that we have r^3 instead of r .

If we further consider the physical meaning of the Stokes' stream function, then it is evident that if we deal with space of five-dimensions, the components of velocity will be given by $-\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial z}$ and $+\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial r}$, since the

circumference of a circle got by rotating a point round the z -axis in space of five-dimensions is $2\pi^2 \cdot r^3$ (see Appendix). The condition for irratational motion will therefore lead to equation 3.

Let us, therefore, consider the irratational motion, with symmetry about an axis, of an incompressible fluid in space of five dimensions.

Let x_1, x_2, x_3, x_4, x_5 be the five co-ordinates, and let ϕ be the velocity potential of the motion. That such a potential exists follows from the assumption that the motion is irratational.

The components of velocity will be given by :

$$\begin{aligned} v_{x_1} &= -\frac{\partial \phi}{\partial x_1} \\ v_{x_2} &= -\frac{\partial \phi}{\partial x_2} \\ &\vdots \\ v_{x_5} &= -\frac{\partial \phi}{\partial x_5} \end{aligned}$$

The condition for the incompressibility of the fluid is

$$\begin{aligned} \frac{\partial v_{x_1}}{\partial x_1} + \frac{\partial v_{x_2}}{\partial x_2} + \dots + \frac{\partial v_{x_5}}{\partial x_5} &= 0; \\ \text{i. e.} \quad \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_2^2} + \dots + \frac{\partial^2 \phi}{\partial x_5^2} &= 0. \end{aligned}$$

If we now choose the x_1 axis as the axis of symmetry, then

$$r^2 = x_2^2 + x_3^2 + x_4^2 + x_5^2,$$

where r is the distance of the point (x_1, x_2, x_3, x_4, x_5) from the x_1 axis.

Now, since there is symmetry about the x_1 -axis, we shall have

$$\phi = \phi(z, r),$$

where in the further development z is taken as the axis of symmetry,

i. e. $z \equiv x_1$.

Again,

$$\begin{aligned} \frac{\partial \phi}{\partial x_2} &= \frac{\partial \phi}{\partial r} \cdot \frac{\partial r}{\partial x_2} \\ \frac{\partial^2 \phi}{\partial x_2^2} &= \frac{\partial^2 \phi}{\partial r^2} \left(\frac{\partial r}{\partial x_2} \right)^2 + \frac{\partial \phi}{\partial r} \cdot \frac{\partial^2 r}{\partial x_2^2} \\ &= \frac{\partial^2 \phi}{\partial r^2} \cdot \frac{x_2^2}{r^2} + \frac{\partial \phi}{\partial r} \left(\frac{1}{r} - \frac{x_2^2}{r^3} \right) \end{aligned}$$

with similar expressions for

$$\frac{\partial^2 \phi}{\partial x_3^2}, \frac{\partial^2 \phi}{\partial x_4^2}, \frac{\partial^2 \phi}{\partial x_5^2}.$$

Adding, we get :

$$\frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \phi}{\partial x_2^2} + \dots + \frac{\partial^2 \phi}{\partial x_5^2} = \frac{\partial^2 \phi}{\partial z^2} + \frac{\partial^2 \phi}{\partial r^2} + \frac{3}{r} \cdot \frac{\partial \phi}{\partial r} = 0.$$

$$i. e. \quad \frac{\partial}{\partial z} \left(r^3 \cdot \frac{\partial \phi}{\partial z} \right) + \frac{\partial}{\partial r} \left(r^3 \cdot \frac{\partial \phi}{\partial r} \right) = 0 \quad - \quad - \quad (4)$$

Further,

$$\left. \begin{aligned} v_z &= - \frac{\partial \phi}{\partial z} \\ v_r &= - \frac{\partial \phi}{\partial r} \end{aligned} \right\} \quad - \quad - \quad (5)$$

where v_z and v_r are the components of velocity parallel to the z and r axis respectively.

Equation (4) may also be directly obtained by considering the flow through an element $\delta z \cdot \delta r$ rotated in the five-dimensional space round the z -axis.

Equation 4 may also be written :

$$\frac{\partial}{\partial z} (r^3 \cdot v_z) + \frac{\partial}{\partial r} (r^3 \cdot v_r) = 0.$$

This is, however, the condition that

$$r^3 v_r \cdot dz - r^3 \cdot v_z \cdot dr$$

may be a complete differential. A function ψ therefore exists such that

$$r^3 v_r = \frac{\partial \psi}{\partial z}, \quad r^3 v_z = \frac{\partial \psi}{\partial r},$$

i. e.

$$\left. \begin{aligned} v_z &= - \frac{1}{r^3} \cdot \frac{\partial \psi}{\partial r} \\ v_r &= + \frac{1}{r^3} \cdot \frac{\partial \psi}{\partial z} \end{aligned} \right\} \quad - \quad - \quad - \quad (6)$$

Since the motion is irrotational, we have :

$$\frac{\partial v_z}{\partial r} - \frac{\partial v_r}{\partial z} = 0,$$

i. e.

$$\frac{\partial}{\partial z} \left(\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial z} \right) + \frac{\partial}{\partial r} \left(\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial r} \right) = 0 \quad - \quad - \quad (7)$$

The stream-lines are given by

$$\frac{dz}{v_z} = \frac{dr}{v_r},$$

i. e.

$$\frac{\partial \psi}{\partial z} \cdot dz + \frac{\partial \psi}{\partial r} \cdot dr = 0,$$

i. e. $\psi = \text{constant}$ is the equation for the stream-lines.

We note, then, that equations 3 and 7 are identical, and, further, that in the hydrodynamical problem $\psi = \text{const.}$ represents the stream-lines, and in the elastic problem $\psi = \text{const.}$ represents the stress-lines.

We shall, therefore, firstly draw up the velocity potentials and stream-functions for certain motions with symmetry about an axis in space of five dimensions. Since the surface of revolution of the body is free from external forces, therefore any of the stream-lines $\psi = \text{const.}$ may be taken

as the boundary lines of the body of revolution in a meridian plane. The stress-components will then be given by

$$\left. \begin{aligned} r\hat{\theta} &= -\frac{1}{r^2} \cdot \frac{\partial \psi}{\partial z} = r \cdot \frac{\partial \phi}{\partial r} \\ \hat{\theta}z &= +\frac{1}{r^2} \cdot \frac{\partial \psi}{\partial r} = r \cdot \frac{\partial \phi}{\partial z} \end{aligned} \right\} \quad (8)$$

since

$$\left. \begin{aligned} v_z &= -\frac{\partial \phi}{\partial z} = -\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial r} \\ v_r &= -\frac{\partial \phi}{\partial r} = +\frac{1}{r^3} \cdot \frac{\partial \psi}{\partial z} \end{aligned} \right\} \quad (9)$$

SECTION 3.

The Displacement u_θ .

For u_θ we have from 2 and 8:

$$\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} = \frac{1}{\mu} \cdot r\hat{\theta} = \frac{r}{\mu} \cdot \frac{\partial \phi}{\partial r}$$

$$\frac{\partial u_\theta}{\partial z} = \frac{1}{\mu} \cdot \hat{\theta}z = \frac{r}{\mu} \cdot \frac{\partial \phi}{\partial z}$$

i. e.

$$\frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) = \frac{1}{\mu} \cdot \frac{\partial \phi}{\partial r}$$

$$\frac{\partial}{\partial z} \left(\frac{u_\theta}{r} \right) = \frac{1}{\mu} \cdot \frac{\partial \phi}{\partial z}$$

i. e.

$$\frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) dr + \frac{\partial}{\partial z} \left(\frac{u_\theta}{r} \right) dz = \frac{1}{\mu} \left(\frac{\partial \phi}{\partial r} \cdot dr + \frac{\partial \phi}{\partial z} \cdot dz \right).$$

Integrating, we have

$$u_\theta = \frac{r}{\mu} (\phi + c) \quad (10)$$

where c is a constant.

μ is the "rigidity" of the material, and

$$\mu = \frac{E}{2(1+\sigma)}$$

where E is "Young's modulus" and σ "Poisson's Ratio."

From equation 10 we see further that $\phi = \text{constant}$ will be the lines of constant angular displacement.

SECTION 4.

Polar Form of Equations.

We make the transformation

$$z = \rho \cos \theta$$

$$r = \rho \sin \theta,$$

and we get for the equations for ϕ and ψ

$$\frac{\partial}{\partial \rho} \left(\rho^4 \cdot \frac{\partial \phi}{\partial \rho} \right) + \rho^2 \cdot \frac{\partial^2 \phi}{\partial \theta^2} + 3\rho^2 \cot \theta \cdot \frac{\partial \phi}{\partial \theta} = 0 \quad (11)$$

$$\frac{\partial}{\partial \rho} \left(\frac{1}{\rho^2} \cdot \frac{\partial \psi}{\partial \rho} \right) + \frac{1}{\rho^4} \cdot \frac{\partial^2 \psi}{\partial \theta^2} - \frac{3 \cot \theta}{\rho^4} \cdot \frac{\partial \psi}{\partial \theta} = 0 \quad (12)$$

For the components of velocity v_ρ and v_θ we have

$$\left. \begin{aligned} v_\rho &= -\frac{\partial \phi}{\partial \rho} = -\frac{1}{\rho^3 \sin^3 \theta} \cdot \frac{1}{\rho} \cdot \frac{\partial \psi}{\partial \theta} \\ v_\theta &= -\frac{\partial \phi}{\rho \partial \theta} = \frac{1}{\rho^3 \sin^3 \theta} \cdot \frac{\partial \psi}{\partial \rho} \end{aligned} \right\} \quad (13)$$

SECTION 5.

Uniform Stream.

For a uniform stream of velocity, v_o parallel to the z -axis we have

$$v_o = -\frac{\partial \phi_o}{\partial z}, \quad o = -\frac{\partial \phi_o}{\partial r},$$

where ϕ_o is the velocity potential of such a motion.

On integrating we get

$$\phi_o = -v_o z = -v_o \rho \cos \theta \quad (14)$$

where the arbitrary constant is taken as zero.

From equations 9 we further have

$$\begin{aligned} v_o &= -\frac{1}{r^3} \cdot \frac{\partial \psi_o}{\partial r} \\ o &= +\frac{1}{r^3} \cdot \frac{\partial \psi_o}{\partial z}, \end{aligned}$$

where ψ_o is the stream-function for this motion.

Integrating, we get

$$\psi_o = \frac{v_o r^4}{4} = -\frac{v_o}{4} \rho^4 \sin^4 \theta \quad (15)$$

The lines $\psi_o = \text{const.}$ are in this case lines parallel to the z -axis, and hence we have the case of a circular cylinder.

From equations 8 the stress-components are given by:

$$\begin{aligned} \widehat{r\theta} &= 0 \\ \widehat{\theta z} &= -v_o r. \end{aligned}$$

From equation 10 the displacement u_θ is given by

$$u_\theta = -\frac{v_o r z}{\mu},$$

where the constant c is taken such that over the plane $z = 0$ the displacement is zero.

If we agree to take an anti-clockwise direction as being positive, then the moment of the couples at the ends will be given by

$$M_z = - \int_0^{2\pi} \int_0^{r_0} r^2 \cdot \hat{\theta}_z \cdot dr \cdot d\theta.$$

$$= \frac{\pi v_0^2 r_0^4}{2}.$$

Therefore when a circular cylinder of radius r_0 is subjected to terminal couples of magnitude

$$- \frac{\pi v_0^2 r_0^4}{2},$$

then the stress-components will be given by

$$\hat{r}\theta = 0$$

$$\hat{\theta}z = v_0 r,$$

and the displacement u_θ by

$$u_\theta = \frac{v_0 r z}{\mu}.$$

This agrees with the result that follows from Saint-Venant's theory.

SECTION 6.

Source or Sink.

Let a point source or sink be situated at the origin of the co-ordinate system. If the total flow in unit time across any closed surface surrounding the origin is $8\pi^2 c$, then we shall call c the strength of the source or sink.

Let ϕ_1 be the velocity potential of the motion, and let a sphere of radius ρ with its centre at the origin be described.

Now since the area of the surface of a sphere in space of fine-dimensions is $\frac{8}{3}\pi^2 \rho^4$ (see Appendix), therefore we have

$$- \frac{8}{3}\pi^2 \rho^4 \cdot \frac{\partial \phi_1}{\partial \rho} = 8\pi^2 c.$$

On integrating we get

$$\phi_1 = \frac{c}{\rho^3} \quad \text{--- (16)}$$

This is the only solution of (11) which is independent of θ .

For a source c is positive and for a sink c is negative.

For the components of velocity we have

$$v_\rho = - \frac{\partial \phi_1}{\partial \rho} = \frac{3c}{\rho^4} = - \frac{1}{\rho^4} \cdot \frac{1}{\sin^3 \theta} \cdot \frac{\partial \psi_1}{\partial \theta}$$

$$v_\theta = - \frac{1}{\rho} \cdot \frac{\partial \phi_1}{\partial \theta} = 0 = + \frac{1}{\rho^3} \cdot \frac{1}{\sin^3 \theta} \cdot \frac{\partial \psi_1}{\partial \rho},$$

where ψ_1 is the stream-function for a source or sink.

For the determination of ψ_1 we therefore have

$$\begin{aligned}\frac{\partial \psi_1}{\partial \rho} &= 0 \\ \frac{\partial \psi_1}{\partial \theta} &= -3c \sin^3 \theta.\end{aligned}$$

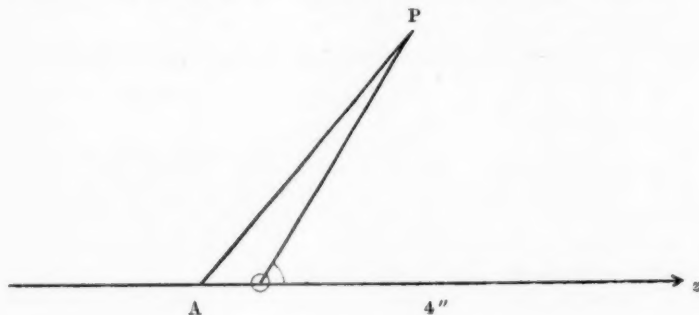
On integrating we get

$$\begin{aligned}\psi &= c (3 \cos \theta - \cos^3 \theta) \\ &= c \cdot \frac{2x^3 + 3xz^2}{(z^2 + r^2)^{\frac{3}{2}}}\end{aligned} \quad (17)$$

SECTION 7.

Doublet.

In considering the motion due to a doublet we must take the doublet on the z -axis, and the axis of the doublet must coincide with the z -axis in order that the resulting motion may be symmetrical about the z -axis.



Let there be a sink at o and an equal source at A , the strength being c . Let P be any point (ρ, θ) .

Now, since the differential equation for the velocity-potential, equation 11, is linear, we find the velocity-potential at P due to the source and sink by superposing the potentials due to each of them.

Hence, if ϕ_2 is the velocity-potential of the doublet, then

$$\begin{aligned}\phi_2 &= \lim_{\delta z \rightarrow 0} \left\{ \frac{c}{(\rho + \delta \rho)^3} - \frac{c}{\rho^3} \right\} \\ &= \lim_{\delta z \rightarrow 0} -\frac{c}{\rho^3} \left\{ \frac{3\delta \rho}{\rho} - 6 \cdot \left(\frac{\delta \rho}{\rho} \right)^2 + \dots \right\} \\ &= \lim_{\delta z \rightarrow 0} \left\{ -\frac{3c \delta z \cos \theta}{\rho^4} + \text{terms of higher degree in } \delta z \right\}\end{aligned}$$

If now we take

$$\int_{\delta z \rightarrow \delta} 3c \, \delta z = \kappa \text{ (finite),}$$

then we get for the velocity potential of a doublet

$$\left. \begin{aligned} \phi_2 &= -\frac{\kappa \cos \theta}{\rho^4} \\ &= -\kappa \frac{z}{(z^2 + r^2)^{\frac{5}{2}}} \end{aligned} \right\} \quad \text{--- (18)}$$

where $\frac{\kappa}{3}$ is the strength of the doublet.

Further, we have :

$$\begin{aligned} v_\rho &= -\frac{\partial \phi_2}{\partial \rho} = -\frac{4\kappa \cos \theta}{\rho^5} = -\frac{1}{\rho^4} \cdot \frac{1}{\sin^3 \theta} \cdot \frac{\partial \psi_2}{\partial \theta} \\ v_\theta &= -\frac{1}{\rho} \cdot \frac{\partial \phi_2}{\partial \theta} = -\frac{\kappa \sin \theta}{\rho^5} = \frac{1}{\rho^3} \cdot \frac{1}{\sin^3 \theta} \cdot \frac{\partial \psi_2}{\partial \rho}, \end{aligned}$$

where ψ_2 is the stream-function of a doublet.

For the determination of ψ_2 we therefore have :

$$\begin{aligned} \frac{\partial \psi_2}{\partial \theta} &= \frac{4\kappa \cos \theta \sin^3 \theta}{\rho} \\ \frac{\partial \psi_2}{\partial \rho} &= -\frac{\kappa \sin^4 \theta}{\rho^3}. \end{aligned}$$

Integrating, we get :

$$\psi_2 = \frac{\kappa \sin^4 \theta}{\rho} \quad \text{--- (19)}$$

Let us now investigate the result obtained by superposing the stream-function of a uniform stream parallel to the z -axis upon that of a doublet. We may do this, since the equation for ψ , equation 12, is linear. If ϕ_3 is the velocity-potential, ψ_3 the stream-function of such a motion, then

$$\begin{aligned} \phi_3 &= \phi_0 + \phi_2 = -v_0 \rho \cos \theta - \frac{\kappa \cos \theta}{\rho^4} \\ \psi_3 &= \psi_0 + \psi_2 = -\frac{v_0}{4} \rho^4 \sin^4 \theta + \frac{\kappa \sin^4 \theta}{\rho}. \end{aligned}$$

1. Case.

If we put $v_0 = 4$, $\kappa = 1$, then

$$\psi_3 = \left(\frac{1}{\rho} - \rho^4 \right) \sin^4 \theta.$$

From this equation we see that $\theta = 0$ and $\rho = 1$ are the stream-lines $\psi = 0$.

To draw the family of curves ψ_3 constant, we may firstly draw the curves

$$\psi_0 = -r^4 = -\rho^4 \sin^4 \theta$$

$$\psi_2 = \frac{\sin^4 \theta}{\rho}$$

for equidistant values of ψ_0 and ψ_2 , and then obtain the curves $\psi_3 = \text{constant}$ by vector addition.

On Plate VIII the family A are the lines $\psi_0 = \text{constant}$, and the family B are the lines $\psi_2 = \text{constant}$. By vector addition we get the family C, whose equation is given by

$$\psi_3 = \left(\frac{1}{\rho} - \rho^4 \right) \sin^4 \theta.$$

Now, any of these stream-lines may be taken as the boundary lines of the body of revolution in a meridian plane, care being, however, taken that the singular points fall outside the body, the singular point being in this case at the origin.

If we take $\psi_3 = 0$ and -1.2 (say), then we shall get a body such as shown in Plate X, the cavity being spherical.

If r_1 is the radius of the body of revolution at $z = +\infty$ and $-\infty$, then r_1 is given by

$$r_1 = \sqrt[4]{-\psi_3}.$$

If r_2 is the radius of the body in the diametral plane $z = 0$, then r_2 is given by the equation

$$r_2^5 + \psi_3 \cdot r_2 - 1 = 0.$$

From the two equations for r_1 and r_2 we find

$$r_2^5 - r_1^4 r_2 - 1 = 0,$$

i. e.

$$\frac{r_2^5}{r_1^5} - \frac{r_2}{r_1} - \frac{1}{r_1^5} = 0.$$

Hence we see that for great values of r_1 , r_2 becomes equal to r_1 .

If, e. g., we take $r_1 = 2$, then $r_2 = 2.015$ approximately. If, therefore, $\psi_3 = -16$ (say) be taken as the outer boundary line of the body of revolution in a meridian plane, then $\psi_3 = -16$ approximates to a straight line parallel to the axis of the body, and hence we get the case of circular cylinder with a spherical "flaw" on its axis.

Again, if we take $\psi_3 = -0.1$ and $\psi_3 = -1.2$ as the boundary lines, then we get a body such as shown in Plate XI, which would correspond to a circular pipe bulged out uniformly along some diametral plane.

For such bodies, then, the stress-components $r\theta$ and θz are given by

$$r\theta = r \frac{\partial \phi_3}{\partial r} = -\frac{1}{r^2} \cdot \frac{\partial \psi_3}{\partial z} = \frac{5zr^2}{(z^2 + r^2)^{\frac{5}{2}}}$$

$$\theta z = r \frac{\partial \phi_3}{\partial z} = +\frac{1}{r^2} \cdot \frac{\partial \psi_3}{\partial r} = -4r + r \cdot \frac{4z^2 - r^2}{(z^2 + r^2)^{\frac{5}{2}}}.$$

Further, from 10 the displacement u_θ will be given by

$$u_\theta = -\frac{zr}{\mu} \left\{ 4 + \frac{1}{(z^2 + r^2)^{\frac{5}{2}}} \right\},$$

where the plane $z = 0$ is taken as the plane of zero displacement.

If r_1 is the radius of the body at $z = +\alpha$ and $-\alpha$, then the moment of the terminal couples is given by

$$M_3 = 2\pi r_1^4.$$

If r_1' and r_1'' are the inner and outer radii at $z = +\alpha$ and $-\alpha$, then

$$M_3' = 2\pi (r_1''^4 - r_1'^4).$$

In general, then, we have. If in a meridian plane the boundaries of the body of revolution are given by

$$-\frac{v_0 r^4}{4} + \frac{\kappa r^4}{(z^2 + r^2)^{\frac{5}{2}}} = \text{const.},$$

then the stress-components are given by

$$\widehat{r\theta} = + \frac{5\kappa z r^2}{(z^2 + r^2)^{\frac{7}{2}}}$$

$$\widehat{\theta z} = -v_0 r + \kappa r \cdot \frac{4z^2 - r^2}{(z^2 + r^2)^{\frac{5}{2}}},$$

and the displacement u_θ by

$$u_\theta = -\frac{zr}{\mu} \left\{ v_0 + \frac{k}{(z^2 + r^2)^{\frac{5}{2}}} \right\}$$

the moment of the terminal couples being

$$\frac{\pi v_0}{2} (r_1''^4 - r_1'^4),$$

where r_1' and r_1'' are the inner and the outer radii of the body at the ends.

2. Case.

If we put $v_0 = 4$ and $k = -1$, i. e. change the axis of the doublet, then

$$\phi_3 = -4z + \frac{z}{(z^2 + r^2)^{\frac{5}{2}}}$$

$$\psi_3 = -r^4 - \frac{r^4}{(z^2 + r^2)^{\frac{5}{2}}}.$$

Proceeding as in the previous case we get the stream-lines C in Plate IX.

In this case we have a circular cylinder which has been bulged in slightly as in Plate XII.

The formulæ for the stress-components, etc., follow from the general formulæ of Case 1 by putting $v_0 = 4$, $k = -1$.

SECTION 8.

Uniform Stream and Two Doublets.

Let us now investigate the motion due to a uniform stream parallel to the z -axis superposed upon that due to two doublets of equal strengths at the points $z = +a$ and $z = -a$.

From 14 and 18 we have for the velocity-potential of such a motion

$$\phi_4 = -v_0 z - \frac{k(z+a)}{\{(z+a)^2 + r^2\}^{\frac{3}{2}}} - \frac{k(z-a)}{\{(z-a)^2 + r^2\}^{\frac{3}{2}}}$$

and from 15 and 19 we have for the stream-function

$$\psi_4 = -\frac{v_0 r^4}{4} + \frac{k r^4}{\{(z+a)^2 + r^2\}^{\frac{5}{2}}} + \frac{k r^4}{\{(z-a)^2 + r^2\}^{\frac{5}{2}}}.$$

Again, if we put $v_0 = 4$, $k = 1$, we have

$$\psi_4 = -r^4 + r^4 \left[\frac{1}{\{(z+a)^2 + r^2\}^{\frac{5}{2}}} + \frac{1}{\{(z-a)^2 + r^2\}^{\frac{5}{2}}} \right].$$

This family of curves may be obtained in the following way:

Firstly, the family of curves given by

$$\psi_4 = \frac{r^4}{(z^2 + r^2)^{\frac{5}{2}}} = \frac{\sin^4 \theta}{\rho}$$

is traced on transparent linen paper (see Plate A). This is now laid on Plate VIII, the two z -axes coinciding, and the doublet of Plate A being distant $2a$ from the origin of Plate VIII. On top of this another strip of linen paper is placed, and on this paper the curves, got by combining the curves C of Plate VIII and the curves of Plate A, are traced. In this way, then, we arrive at the stream-lines due to the uniform stream and the two doublets.

The stream-lines $\psi_4 = 0$ are given by $r = 0$ and

$$\frac{1}{\{(z+a)^2 + r^2\}^{\frac{5}{2}}} + \frac{1}{\{(z-a)^2 + r^2\}^{\frac{5}{2}}} = 1.$$

This is a closed curve with symmetry about the z and r axes, and if this is taken as one of the boundary lines of the body of revolution in a meridian plane, then again we have the case of a circular cylinder with a "flaw" on its axis, the dimensions of the "flaw" being small compared to the radius of the cylinder. By the above method this curve has been obtained for $a = \frac{1}{2}$ (see Plate B). From this we get a body such as shown in Plate XIII.

In such bodies the stress-components will be given by

$$\begin{aligned} r\theta &= r \cdot \frac{\partial \phi_4}{\partial r} = 5kr^2 \left\{ \frac{z+a}{\{(z+a)^2 + r^2\}^{\frac{5}{2}}} + \frac{z-a}{\{(z-a)^2 + r^2\}^{\frac{5}{2}}} \right\} \\ \theta z &= r \cdot \frac{\partial \phi_4}{\partial z} = -v_0 r + kr \left\{ \frac{4(z+a)^2 - r^2}{\{(z+a)^2 + r^2\}^{\frac{5}{2}}} - \frac{4(z-a)^2 - r^2}{\{(z-a)^2 + r^2\}^{\frac{5}{2}}} \right\}. \end{aligned}$$

The displacement u_θ is given by

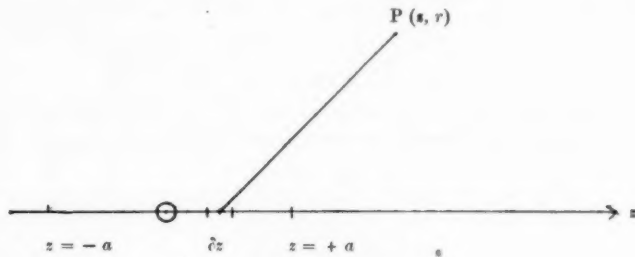
$$u_\theta = -\frac{r}{\mu} \left\{ v_0 z + \frac{k(z+a)}{\{(z+a)^2 + r^2\}^{\frac{3}{2}}} + \frac{k(z-a)}{\{(z-a)^2 + r^2\}^{\frac{3}{2}}} \right\}.$$

The moment of the terminal couple will again be $+\frac{\pi v_0 r_1^4}{2}$, where r_1 is the radius of the body at $z = \pm \infty$.

SECTION 9.

Line Source.

Let λ be the linear intensity of a uniform and continuous distribution of sources along the z -axis from $z = -a$ to $z = +a$.



The velocity-potential due to an element δz_1 is

$$\delta\phi_5 = \lambda \cdot \frac{\delta z_1}{\{(z - z_1)^2 + r^2\}^{\frac{3}{2}}}$$

at the point (z, r) .

The velocity-potential due to the whole distribution is therefore

$$\phi_5 = \lambda \int_{-a}^{+a} \frac{dz_1}{\{(z - z_1)^2 + r^2\}^{\frac{3}{2}}}.$$

Putting $z - z_1 = r \tan \zeta$, we get

$$\begin{aligned} \phi_5 &= -\frac{\lambda}{r^2} \int \cos \zeta \cdot a \, \zeta \\ &\quad + \tan^{-1} \frac{z+a}{r} \\ &= \frac{\lambda}{r^2} \left\{ \frac{z+a}{\{(z+a)^2 + r^2\}^{\frac{1}{2}}} - \frac{z-a}{\{(z-a)^2 + r^2\}^{\frac{1}{2}}} \right\}. \end{aligned}$$

From this we get

$$\frac{\partial \phi_5}{\partial z} = \lambda \left\{ \frac{1}{\{(z+a)^2 + r^2\}^{\frac{3}{2}}} - \frac{1}{\{(z-a)^2 + r^2\}^{\frac{3}{2}}} \right\}$$

$$\frac{\partial \phi_5}{\partial r} = -\frac{\lambda}{r^3} \left\{ (z+a) \cdot \frac{2(z+a)^2 + 3r^2}{\{(z+a)^2 + r^2\}^{\frac{3}{2}}} - (z-a) \cdot \frac{2(z-a)^2 + 3r^2}{\{(z-a)^2 + r^2\}^{\frac{3}{2}}} \right\}.$$

From 9 we have

$$\frac{\partial \phi_5}{\partial z} = + \frac{1}{r^3} \cdot \frac{\partial \psi_5}{\partial r}$$

$$\frac{\partial \phi_5}{\partial r} = - \frac{1}{r^3} \cdot \frac{\partial \psi_5}{\partial z},$$

where ψ_5 is the stream-function of the motion.

Substituting the values of $\frac{\partial \phi_5}{\partial z}$, $\frac{\partial \phi_5}{\partial r}$, and integrating, we get

$$\psi_5 = \lambda \left\{ \frac{2(z+a)^2 + r^2}{\{(z+a)^2 + r^2\}^{\frac{1}{2}}} - \frac{2(z-a)^2 + r^2}{\{(z-a)^2 + r^2\}^{\frac{1}{2}}} \right\}.$$

Each of the terms of this equation satisfies the differential equation for ψ . We again firstly draw the two families of curves

$$\psi'_5 = \lambda \cdot \frac{2(z+a)^2 + r^2}{\{(z+a)^2 + r^2\}^{\frac{1}{2}}}$$

$$\psi''_5 = \lambda \cdot \frac{2(z-a)^2 + r^2}{\{(z-a)^2 + r^2\}^{\frac{1}{2}}},$$

and from these two families of curves find

$$\psi_5 = \psi'_5 - \psi''_5.$$

If we transfer the origin to the point $z = -a$, and then express the equation in the polar form, then we have

$$\psi'_5 = \frac{\lambda \rho}{2} (\cos 2\theta + 3).$$

Drawing this family of curves with the origin at (i) $z = -a$, (ii) $z = +a$, and combining the two families by the vector law, we find the stream-lines for a uniform line distribution of sources.

On Plate XIV the families A and B are given by

$$\psi'_5 = \frac{\rho}{2} (\cos 2\theta + 3) \quad (\lambda = 1)$$

with the origin at $z = +\frac{1}{2}$ and $z = -\frac{1}{2}$ respectively. Combining these, we find the family C, which will then be the stream-lines for a line source of strength $\lambda = 1$.

Superposing upon this, the uniform stream given by

$$\psi_0 = -16r^4 \quad (v_0 = 64)$$

we find the stream-lines E.

Taking the stream-line $\psi_5 = -2$ as one of the boundary lines of the body of revolution in a meridian plane we get a body such as shown on Plate XV.

If $z \rightarrow \pm \alpha$, the stress-lines become parallel to the axis of the body, and if $\psi_5 = \bar{\psi}_5$ be a definite stress-line, then its distance r_1 from the axis at $z = +\alpha$ is given by

$$\bar{\psi}_5 = -\frac{v_0 r_1^4}{4} + 4a\lambda,$$

i. e.

$$r_1 = \sqrt[4]{\frac{-4\bar{\psi}_5 + 16a\lambda}{v_0}}.$$

Again, for $z = -\alpha$ we have

$$\psi_5 = -\frac{v_0 r_2^4}{4} - 4a\lambda,$$

where r_2 is the distance of the stress-line $\bar{\psi}_5 = \text{constant}$ from the axis at $z = -\alpha$. This gives

$$r_2 = \sqrt[4]{\frac{-4\bar{\psi}_5 - 16a\lambda}{v_0}}.$$

In the special case, where we took $v_0 = 64$, $\lambda = 1$, we have

$$r_1 = \sqrt[4]{\frac{-\bar{\psi}_5 + 2}{2}}$$

$$r_2 = \sqrt[4]{\frac{-\bar{\psi}_5 - 2}{2}}.$$

For $\psi_5 = -2$, $r_1 = .707$, $r_2 = 0$, which corresponds with the value got from Plate XIV.

Further, we note that if a sufficiently great negative value is taken for ψ_5 , then the stream-line approximates to a straight line parallel to the z -axis, and hence we get the case of a circular cylinder with a hole bored in at the one end, the equation of the hole in a meridian plane being given by $\psi_5 = -2$.

If $\psi_5 = -62$, $r_1 = 1.414$ and $r_2 = 1.392$, the difference between the radii then being .022.

In such a body, then, the stress-components are given by

$$r\theta = r \cdot \frac{\partial \psi_5}{\partial r} = \frac{\lambda}{r^2} \left\{ (z-a) \cdot \frac{2(z-a)^2 + 3r^2}{\{(z-a)^2 + r^2\}^{\frac{3}{2}}} - (z+a) \cdot \frac{2(z+a)^2 + 3r^2}{\{(z+a)^2 + r^2\}^{\frac{3}{2}}} \right\}$$

$$\theta z = r \cdot \frac{\partial \phi_5}{\partial z} = -v_0 r + \lambda r \left\{ \frac{1}{\{(z+a)^2 + r^2\}^{\frac{3}{2}}} - \frac{1}{\{(z-a)^2 + r^2\}^{\frac{3}{2}}} \right\}.$$

The displacement u_θ is given by

$$u_\theta = \frac{r}{\mu} (\phi + c).$$

If we now take plane $z = -b$ as being fixed, where b is great compared to a , such that quantities of the order $\frac{a}{b^3}$ and higher may be neglected, then the displacement is given by

$$u_\theta = -\frac{v_0(z+C)r}{\mu} + \frac{\lambda}{\mu r} \left\{ \frac{z+a}{\{(z+a)^2 + r^2\}^{\frac{3}{2}}} - \frac{z-a}{\{(z-a)^2 + r^2\}^{\frac{3}{2}}} \right\}.$$

The moment of the terminal couples is $\frac{\pi v_0 r_1^4}{2}$, where r_1 is the radius of the body at $z = -\alpha$.

SECTION 10.

Ring Source.

We take the ring source such that there shall be symmetry about the z -axis, the plane of the ring passing through the origin.

In order to get the velocity-potential of such a ring source, we may firstly calculate the potential due to a uniform and continuous distribution of sources over a circular plate perpendicular to the z -axis, and then differentiate this potential with respect to the radius, which will give us the potential due to a ring.

Let the co-ordinates of any point on the plate be given by $(o, x'_2, x'_3, x'_4, x'_5)$ and that of the point P, where the potential is to be found, (z, x_2, x_3, x_4, x_5) .

The velocity-potential due to an element $\delta x'_2 \times \delta x'_3 \times \delta x'_4 \times \delta x'_5$ is

$$\delta \phi_6 = c_1 \cdot \frac{\delta x'_2 \cdot \delta x'_3 \cdot \delta x'_4 \cdot \delta x'_5}{\{z^2 + (x_2 - x'_2)^2 + (x_3 - x'_3)^2 + (x_4 - x'_4)^2 + (x_5 - x'_5)^2\}^{\frac{3}{2}}},$$

where c_1 is the strength of the distribution.

For a circular plate of radius a_1 the potential will therefore be

$$\phi_6 = c_1 \iiint \frac{dx'_2, dx'_3, dx'_4, dx'_5}{\{z^2 + (x_2 - x'_2)^2 + \dots + (x_5 - x'_5)^2\}^{\frac{3}{2}}},$$

integrated over the region

$$x_2'^2 + x_3'^2 + x_4'^2 + x_5'^2 \leq a_1^2.$$

Now, by applying linear transformations we readily see that, without the loss of generality, we may choose our axes such that the co-ordinates of the point P are (z, x_2, o, o, o) .

The result then reduces to:

$$\phi_6 = c_2 \iiint \frac{dx'_2, dx'_3, dx'_4, dx'_5}{\{z^2 + (x_2 - x'_2)^2 + x_3'^2 + x_4'^2 + x_5'^2\}^{\frac{3}{2}}},$$

$$(x_2'^2 + x_3'^2 + x_4'^2 + x_5'^2 \leq a_1^2).$$

We now apply the transformation,

$$\begin{aligned}x'_2 &= a \cos a_1, \\x'_3 &= a \sin a_1 \cos a_2, \\x'_4 &= a \sin a_1 \sin a_2 \cos a_3, \\x'_5 &= a \sin a_1 \sin a_2 \sin a_3,\end{aligned}$$

which we may call a four-dimensional polar transformation.

The integral is then transformed into

$$\phi_6 = c_1 \iiint \frac{\partial(x'_2, x'_3, x'_4, x'_5)}{\partial(a, a_1, a_2, a_3)} \cdot da \cdot da_1 \cdot da_2 \cdot da_3, \\ \{z^2 + a^2 + r^2 - 2ar \cos a_1\}^{\frac{3}{2}},$$

where $r = x_2$, and the Jacobian

$$\frac{\partial(x'_2, x'_3, x'_4, x'_5)}{\partial(a, a_1, a_2, a_3)} = \begin{vmatrix} \frac{\partial x'_2}{\partial a} & \frac{\partial x'_2}{\partial a_1} & \frac{\partial x'_2}{\partial a_2} & \frac{\partial x'_2}{\partial a_3} \\ \frac{\partial x'_3}{\partial a} & - & - & - \\ - & - & - & - \\ \frac{\partial x'_5}{\partial a} & - & - & - \end{vmatrix} \\ = a^3 \sin^2 a_1 \sin a_2.$$

The limits of the above integral may immediately be got by considering that the transformation must be a single-valued one. If we take a to range from 0 to a_1 , then a_1 ranges from 0 to π , a_2 from 0 to π , a_3 from 0 to 2π .

We therefore have

$$\begin{aligned}\phi_6 &= c_1 \int_{a=0}^{a_1} \int_{a_1=0}^{\pi} \int_{a_2=0}^{\pi} \int_{a_3=0}^{2\pi} \frac{a^3 \sin^2 a_1 \sin a_2 \cdot da \cdot da_1 \cdot da_2 \cdot da_3}{\{z^2 + a^2 + r^2 - 2ar \cos a_1\}^{\frac{3}{2}}}, \\ &= 4\pi c_1 \int_{a=0}^{a_1} \int_{a_1=0}^{\pi} \frac{a^3 \sin^2 a_1 \cdot da \cdot da_1}{\{z^2 + a^2 + r^2 - 2ar \cos a_1\}^{\frac{3}{2}}}.\end{aligned}$$

Now the rate of variation of this potential with respect to the radius will be given by

$$\frac{\partial \phi_6}{\partial a} = 4\pi a^3 c_1 \int_{a_1=0}^{\pi} \frac{\sin^2 a_1 \cdot da_1}{\{z^2 + a^2 + r^2 - 2ar \cos a_1\}^{\frac{3}{2}}}$$

and therefore the potential of a ring element of radius a and breadth δa will be given by

$$\phi'_6 = 4\pi a^3 c_1 \cdot \delta a \int_{a_1=0}^{\pi} \frac{\sin^2 a_1 \cdot da_1}{\{z^2 + a^2 + r^2 - 2ar \cos a_1\}^{\frac{3}{2}}}.$$

Or if we go over to the limit for $\delta a = 0$, and take $\lim_{\delta a \rightarrow 0} \frac{c_1 \delta a}{\delta a} = c$ where c is finite, then we get for the velocity-potential of a ring source of radius a

$$\phi'_0 = 4\pi a^3 c \int_{a_1=0}^{\pi} \frac{\sin^2 a_1 \cdot da_1}{\{z^2 + r^2 + a^2 - 2ar \cos a_1\}^{\frac{3}{2}}}.$$

Putting $\pi - a = 2\chi$, the result reduces to

$$\begin{aligned} \phi'_0 &= 8\pi a^3 c \int_0^{\frac{\pi}{2}} \frac{\sin^2 2\chi \cdot d\chi}{\{z^2 + r^2 + a^2 + 2ar \cos 2\chi\}^{\frac{3}{2}}} \\ &= \frac{8\pi a^3 c}{\{z^2 + (a+r)^2\}^{\frac{3}{2}}} \int_0^{\frac{\pi}{2}} \frac{\sin^2 2\chi \cdot d\chi}{\Delta^3 \chi}, \end{aligned}$$

where $k^2 = \frac{4ar}{z^2 + (a+r)^2}$, and $\Delta \chi = \sqrt{1 - k^2 \sin^2 \chi}$.

Let us now consider the integral

$$I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin^2 2\chi \cdot a\chi}{\Delta^3 \chi}.$$

We have the identity:

$$\begin{aligned} \frac{d}{d\chi} \left(\frac{\sin \chi \cos \chi}{\Delta \chi} \right) &= \frac{k^2 \sin^2 \chi \cos^2 \chi}{\Delta^3 \chi} + \frac{\cos^2 \chi}{\Delta \chi} - \frac{\sin^2 \chi}{\Delta \chi} \\ &= \frac{k^2}{4} \cdot \frac{\sin^2 2\chi}{\Delta^3 \chi} + \frac{1}{\Delta \chi} - \frac{2 \sin^2 \chi}{\Delta \chi}. \end{aligned}$$

Integrating between the limits 0 and χ , we get:

$$\int_0^{\chi} \frac{\sin^2 2\chi \cdot d\chi}{\Delta^3 \chi} = \frac{4 \sin \chi \cos \chi}{k^2 \Delta \chi} + \frac{8}{k^2} \int_0^{\chi} \frac{\sin^2 \chi \cdot d\chi}{\Delta \chi} - \frac{4}{k^2} \int_0^{\chi} \frac{\chi \cdot d\chi}{\Delta \chi}.$$

$$\text{But } \int_0^{\chi} \frac{\sin^2 \chi}{\Delta \chi} \cdot a\chi = \frac{1}{k^2} \int_0^{\chi} \frac{a\chi}{\Delta \chi} - \frac{1}{k^2} \int_0^{\chi} \Delta \chi \cdot a\chi = \frac{F(\chi) - E(\chi)}{k^2},$$

where $F(\chi)$ and $E(\chi)$ are Legendre's elliptic integrals of the first and second species respectively.

We therefore have

$$\int_0^{\chi} \frac{\sin^2 2\chi \cdot d\chi}{\Delta^3 \chi} = \frac{2 \sin 2\chi}{\Delta \chi} + \frac{4}{k^4} \{ (2 - k^2) F(\chi) - 2E(\chi) \}.$$

For the amplitude $\chi = \frac{\pi}{2}$, we therefore have

$$I_1 = \int_0^{\frac{\pi}{2}} \frac{\sin^2 2\chi}{\Delta^3 \chi} \cdot d\chi = \frac{4}{k^4} \{ (2 - k^2) K - 2E \} \quad (20)$$

where K and E are the complete elliptic integrals of the first and second species respectively.

Substituting this value of I_1 in the expression for ϕ'_6 , we get

$$\phi'_6 = \frac{3\pi a r^3 c}{k^4 \{z^2 + (a+r)^2\}^{\frac{3}{2}}} \{ (2 - k^2) K - 2E \},$$

where

$$k^2 = \frac{4ar}{z^2 + (a+r)^2} \quad (21)$$

Substituting the value of $z^2 + (a+r)^2$ from the last equation, we have

$$\phi'_6 = \frac{4\pi c}{k} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} \{ (2 - k^2) K - 2E \} \quad (22)$$

Instead of finding the stream-function ψ'_6 by means of equations 9 and 22, we shall rather find it by making direct use of the stream-function for a point source.

From equation 17 we have for a point source at the origin

$$\psi_0 = c_1 z \cdot \frac{2z^2 + 3r^2}{(z^2 + r^2)^{\frac{3}{2}}}.$$

Hence the stream-function at the point $(z, x_2, 0, 0, 0)$ due to an element $\delta x'_3, \delta x'_3, \delta x'_4, \delta x'_5$ at the point $(0, x'_3, x'_3, x'_4, x'_5)$ will be given by (as in the case of ϕ, x_3, x_4, x_5 may, without the loss of generality, be taken zero):

$$\delta\psi_6 = c_1 z \frac{2z^2 + 3 \{ (x_2 - x'_2)^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2 \} dx'_3 dx'_4 dx'_5}{\{ z^2 + (x_2 - x'_2)^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2 \}^{\frac{3}{2}}}.$$

Hence, for a circular plate of radius a_1 we shall have

$$\psi_6 = c_1 z \iiint \frac{2z^2 + 3 \{ (x'_3 - x'_2)^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2 \} dx'_3 dx'_4 dx'_5}{\{ z^2 + (x_2 - x'_2)^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2 \}^{\frac{3}{2}}} \\ (x'_3{}^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2 \leq a_1^2).$$

By applying the four-dimensional polar transformation the integral is transformed into

$$\psi_6 = c_1 z \int_{a=0}^{a_1} \int_{a_1=0}^{\pi} \int_{a_2=0}^{\pi} \int_{a_3=0}^{2\pi} \frac{a^3 \sin^2 a_1 \sin a_2 \{ 2z^2 + 3 [a^2 + r^2 - 2ar \cos a_1] \} da da_1 da_2 da_3}{\{ z^2 + a^2 + r^2 - 2ar \cos a_1 \}^{\frac{3}{2}}} \\ = 4\pi c_1 z \int_{a=0}^{a_1} \int_{a_1=0}^{\pi} \frac{a^3 \sin^2 a_1 \{ 2z^2 + 3 [a^2 + r^2 - 2ar \cos a_1] \} da da_1}{\{ z^2 + a^2 + r^2 - 2ar \cos a_1 \}^{\frac{3}{2}}}$$

where $r \equiv x_2$.

For a ring source of radius a the stream-function will therefore be

$$\psi'_6 = 4\pi c a^3 z \int_{a_1=0}^{\pi} \frac{\sin^2 a_1 \{ 2z^2 + 3 (a^2 + r^2) - 6ar \cos a_1 \} da_1}{\{ z^2 + a^2 + r^2 - 2ar \cos a_1 \}^{\frac{3}{2}}} \\ = 4\pi c a^3 z \{ [2z^2 + 3 (a^2 + r^2)] \int_0^{\pi} \frac{\sin^2 a_1 da_1}{[z^2 + \dots - 2ar \cos a_1]^{\frac{3}{2}}} - 6ar \\ \int_0^{\pi} \frac{\sin^2 a_1 \cos a_1 da_1}{[z^2 + \dots - 2ar \cos a_1]^{\frac{3}{2}}} \}$$

Put $\pi - a_1 = 2\chi$, and we get:

$$\psi_6 = \frac{8\pi ca^3 z}{[z^2 + (a+r)^2]} \left\{ [2z^2 + 3(a^2 + r^2)] \int_0^{\frac{\pi}{2}} \frac{\sin^2 2\chi}{\Delta^3 \chi} d\chi + 6ar \int_0^{\frac{\pi}{2}} \frac{\sin^2 2\chi \cos 2\chi}{\Delta^3 \chi} \cdot d\chi \right\}$$

Let us now consider the two integrals

$$\begin{aligned} I_1 &= \int_0^{\frac{\pi}{2}} \frac{\sin^2 2\chi}{\Delta^3 \chi} \cdot d\chi \\ &= \frac{4}{k^4} \{ (2 - k^2) K - 2E \} \text{ from equation 20.} \end{aligned}$$

$$\begin{aligned} I_2 &= \int_0^{\frac{\pi}{2}} \frac{\sin^2 2\chi \cdot \cos 2\chi}{\Delta^3 \chi} \cdot d\chi \\ &= \int_0^{\frac{\pi}{2}} \frac{\sin^2 2\chi - 8 \sin^4 \chi \cos^2 \chi}{\Delta^3 \chi} \cdot d\chi \\ &= I_1 - 8I_3, \end{aligned}$$

where

$$I_3 = \int_0^{\frac{\pi}{2}} \frac{\sin^4 \chi \cos^2 \chi}{\Delta^3 \chi} \cdot d\chi.$$

For the evaluation of I_3 we note that

$$\frac{d}{d\chi} \left(\frac{\sin^3 \chi \cos \chi}{\Delta \chi} \right) = \frac{k^2 \sin^4 \chi \cos^2 \chi}{\Delta^3 \chi} + \frac{3 \sin^2 \chi}{\Delta \chi} - \frac{4 \sin^4 \chi}{\Delta \chi}.$$

Integrating, we get:

$$k^2 \int_0^{\frac{\pi}{2}} \frac{\sin^4 \chi \cos^2 \chi}{\Delta^3 \chi} \cdot d\chi = 4 \int_0^{\frac{\pi}{2}} \frac{\sin^4 \chi}{\Delta \chi} \cdot d\chi - 3 \int_0^{\frac{\pi}{2}} \frac{\sin^2 \chi}{\Delta \chi} \cdot d\chi.$$

Again, for the reduction of

$$\int_0^{\frac{\pi}{2}} \frac{\sin^4 \chi}{\Delta \chi} d\chi$$

we have by integrating the identity

$$\frac{d\chi}{d} (\sin \chi \cos \chi \cdot \Delta \chi) = 3k^2 \cdot \frac{\sin^4 \chi}{\Delta \chi} - 2(1 + k^2) \frac{\sin^2 \chi}{\Delta \chi} + \frac{1}{\Delta \chi}.$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin^4 \chi}{\Delta \chi} d\chi = \frac{2(1 + k^2)}{3k^2} \int_0^{\frac{\pi}{2}} \frac{\sin^2 \chi}{\Delta \chi} - \frac{1}{3k^2} \cdot K.$$

But

$$\int_0^{\frac{\pi}{2}} \frac{\sin^2 \chi}{\Delta \chi} \cdot d\chi = \frac{1}{k^2} (K - E);$$

$$\therefore k^2 \int_0^{\frac{\pi}{2}} \frac{\sin^4 \chi \cos^2 \chi}{\Delta^3 \chi} = \frac{8 - k^2}{3k^2} \int_0^{\frac{\pi}{2}} \frac{\sin^2 \chi}{\Delta \chi} \cdot d\chi - \frac{4}{3k^2} \cdot K$$

$$\begin{aligned}
 &= \frac{(8-k^2)}{3k^4} (K-E) - \frac{4}{3k^2} \cdot K \\
 &= \frac{(8-5k^2) K - (8-k^2) E}{3k^4}; \\
 \therefore I_2 &= \frac{(8-5k^2) K - (8-k^2) E}{3k^6}.
 \end{aligned}$$

Substituting in the value for I_2 , we have

$$I_2 = I_1 - \frac{8}{3} \frac{(8-5k^2) K - (8-k^2) E}{k^6}.$$

We therefore find for the stream-function of a ring-source:

$$\begin{aligned}
 \psi'_6 &= \frac{8\pi c a^3 z}{[z^2 + (a+r)^2]^{\frac{3}{2}}} \left\{ [2z^2 + 3(a^2 + r^2)] I_1 + 6ar \left[I_1 - \frac{8}{3} \cdot \frac{(8-5k^2) K - (8-k^2) E}{k^6} \right] \right\} \\
 &= \frac{8\pi c a^3 z}{[z^2 + (a+r)^2]^{\frac{3}{2}}} \left\{ [2z^2 + 3(a+r)^2] I_1 - 16ar \cdot \frac{(8-5k) K - (8-k^2) E}{k^6} \right\}
 \end{aligned}$$

Putting in the value of I_1 , we get

$$\psi'_6 = 4\pi c \cdot \frac{z}{k^3} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} \left\{ f_1(z, r) K + f_2(z, r) E \right\} \quad (23)$$

where $f_1(z, r) = k^2(2-k^2)[2z^2 + 3(a+r)^2] - 4ar(8-5k^2)$

$$f_2(z, r) = 4ar(8-k^2) - 2k^2[2z^2 + 3(a+r)^2].$$

Differentiating 22 with respect to r and z respectively, and substituting the values of $\frac{\delta k}{\delta r}$, $\frac{\delta k}{\delta z}$, $\frac{\partial K}{\partial k}$, $\frac{\partial E}{\partial k}$, given by the equations:

$$\begin{aligned}
 \frac{\delta k}{\delta r} &= \frac{1}{8} k^3 \cdot \frac{z^2 + a^2 - r^2}{ar^2} \\
 \frac{\delta k}{\delta z} &= -\frac{1}{4} k^3 \cdot \frac{z}{ar} \\
 \frac{\partial K}{\partial k} &= \frac{1}{k(1-k^2)} \{ E - (1-k^2) K \} \\
 \frac{\partial E}{\partial k} &= \frac{1}{k} \{ E - K \},
 \end{aligned}$$

we get for the components of velocity due to a ring-source the expressions:

$$v_r = -\frac{\partial \psi'_6}{\partial r} = -\frac{4\pi c}{kr} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} \{ A_1 E - A_2 K \},$$

where

$$A_1 = \frac{1-k^4}{k^2} \cdot \frac{z^2 + a^2 - r^2}{8ar} + 3$$

$$A_2 = k^2 \cdot \frac{z^2 + a^2 - r^2}{4ar} + \frac{3}{2} (1+k^2),$$

k' being the complementary modulus,

$$v_z = -\frac{\partial \psi'_6}{\partial z} = -\frac{\pi c}{r} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} \cdot \frac{kz}{a} \left\{ 2K - \frac{1+k'^2}{k'^2} E \right\}.$$

The velocity along the z -axis is given by

$$(v_z)_{r=0} = -L \frac{\partial \phi'_6}{\partial z}.$$

Writing now,

$$K = \frac{\pi}{2} \left\{ 1 + \frac{1}{4}k^2 + \frac{9}{64}k^4 + \dots \right\} \quad (k^2 < 1)$$

$$E = \frac{\pi}{2} \left\{ 1 - \frac{1}{4}k^2 - \frac{3}{64}k^4 \dots \right\} \quad (k^2 < 1)$$

where $k^2 = \frac{4ar}{z^2 + (a+r)^2}$, and going over to the limit for $r = 0$, we find

$$(v_z)_{r=0} = 6\pi^2 \cdot a^3 c \cdot \frac{z}{[z^2 + a^2]^{\frac{5}{2}}}.$$

At $z = 0$, $(v_z)_{r=0}$ is a maximum at $z = \pm \frac{a}{2}$, where

the velocity is equal to $\pm \frac{3 \times 2^5 \times \pi^2 c}{a \cdot 5^{\frac{5}{2}}}$.

Let us again superpose upon this motion a uniform stream parallel to the z -axis, the velocity of the stream being v_0 . The resultant velocity-potential will be given by

$$\bar{\phi}_6 = \phi_0 + \phi'_6,$$

where the values of ϕ_0 and ϕ'_6 are given by equations 14 and 22.

If we choose $v_0 = \frac{3 \times 2^5 \pi^2 c}{a 5^{\frac{5}{2}}}$, then the stream-lines for such a motion will approximately be as shown in Plate XVI, fig. 1. From this, then, we obtain a body such as shown in Fig. 2, in which the stress-components will be given by

$$\begin{aligned} r\theta &= \frac{4\pi c}{k} \cdot \left(\frac{a}{r}\right)^{\frac{5}{2}} \{A_1 E - A_2 K\} \\ \theta z &= -v_0 r + \pi c \left(\frac{a}{r}\right)^{\frac{3}{2}} \cdot \frac{kz}{a} \left\{2K - \frac{1+k^2}{k^2} E\right\}. \end{aligned}$$

The displacement u_θ is given by

$$u_\theta = \frac{r}{u} \left\{ -v_0(z+b) + \frac{4\pi c}{k} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} \left\{ (2-k^2)K - 2E \right\} \right\},$$

where the plane $z = -b$ is taken as the plane of zero displacement, b being great compared to a . The moment of the terminal couples is $\frac{\pi v_0 r_1^4}{2}$, where r_1 the radius of the body at $z = \pm \infty$.

The equation of the boundary lines of the body in a meridian plane will be given by

$$\psi'_6 - \frac{v_0 r^4}{4} = \text{constant},$$

where ψ'_6 is given by equation 23.

SECTION 11.

Ring Doublet.

Let a circular plate perpendicular to the z -axis, its plane passing through the origin, be uniformly and continuously distributed with doublets having their axes perpendicular to the plane of the plate. If κ_1 is the strength of the distribution, then the velocity-potential of an element $\delta x'_2 \delta x'_3 \delta x'_4 \delta x'_5$, whose co-ordinates are $(o, x'_2, x'_3, x'_4, x'_5)$ at a point (z, x_2, o, o, o) will be given from equation 18 by

$$\delta \phi_7 = -\frac{\kappa_1 \cos \theta}{\rho^2} \cdot \delta x'_2 \delta x'_3 \delta x'_4 \delta x'_5,$$

where θ is the angle between the axes of the doublets and the line joining the points $(o, x'_2, x'_3, x'_4, x'_5)$ and (z, x_2, o, o, o) , and ρ the distance between these points.

Now $\rho^2 = z^2 + (x_2 - x'_2)^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2$, and the direction-cosines of this line are given by

$$\frac{\lambda_1}{z_1} = \frac{\lambda_2}{x_2 - x'_2} = \frac{\lambda_3}{-x'_3} = \frac{\lambda_4}{-x'_4} = \frac{\lambda_5}{-x'_5} = \frac{1}{\rho}.$$

The direction-cosines of the axes of the doublets are 1, o, o, o, o .

Hence $\cos \theta = \frac{z}{\rho}$.

$$\therefore \delta \phi_7 = -\kappa_1 z \cdot \frac{\delta x'_2 \delta x'_3 \delta x'_4 \delta x'_5}{\{z^2 + (x_2 - x'_2)^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2\}^{\frac{3}{2}}}$$

Hence the velocity-potential due to a distribution over a circular plate of radius a_1 will be given by

$$\begin{aligned} \phi_7 &= -\kappa_1 z \iiint \iiint \frac{\delta x'_2 \delta x'_3 \delta x'_4 \delta x'_5}{\{z^2 + (x_2 - x'_2)^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2\}^{\frac{3}{2}}} \\ &= \frac{1}{2} \kappa_1 \frac{\partial}{\partial z} \iiint \iiint \iiint \frac{\delta x'_2 \delta x'_3 \delta x'_4 \delta x'_5}{\{z^2 + (x_2 - x'_2)^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2\}^{\frac{1}{2}}} \end{aligned}$$

where the integration is extended over the finite region

$$x'_2{}^2 + x'_3{}^2 + x'_4{}^2 + x'_5{}^2 \leq a_1^2$$

and in which region the function integrated has no singularities provided $z \neq o$ in the region $x_2 \leq a_1$.

From the above we see, then, that the velocity-potential of a ring doublet is given by

$$\phi'_7 = \frac{\kappa}{c} \cdot \frac{\partial}{\partial z} (\phi'_6),$$

where ϕ'_6 is the velocity-potential due to a ring source.

We therefore have for the velocity-potential of a ring doublet of radius a

$$\phi_7 = \frac{\pi \kappa}{r} \left(\frac{a}{r} \right)^{\frac{1}{2}} \cdot \frac{kz}{a} \left(2K - \frac{1 + k'^2}{k^2} E \right) \quad (24)$$

The components of velocity will be given by

$$v_z = -\frac{\partial \phi'_7}{\partial z} = -\frac{\pi\kappa}{r^2} \left(\frac{a}{r}\right)^{\frac{1}{2}} \cdot k \{B_1 K + B_2 E\},$$

where

$$\left. \begin{aligned} B_1 &\equiv 2 - \frac{z^2}{4ar} \cdot \frac{k^2(1+k^2)}{k'^2} \\ B_2 &\equiv -\frac{1+k'^2}{k'^2} + \frac{z^2}{2ar} \cdot k^2 \left(1 + \frac{k^2}{k'^4}\right) \end{aligned} \right\} \quad (25)$$

$$v_r = -\frac{\partial \phi'_7}{\partial r} = -\frac{\pi\kappa}{2ar^2} \cdot \left(\frac{a}{r}\right)^{\frac{1}{2}} \cdot kz \{C_1 K + C_2 E\}.$$

$$\left. \begin{aligned} C_1 &\equiv \frac{k^2(1+k^2)}{k'^2} \cdot \frac{z^2 + a^2 - r^2}{4ar} - 10 \\ C_2 &\equiv 5 \cdot \frac{1+k'^2}{k'^2} - k^2 \left(1 + \frac{k^2}{k'^4}\right) \cdot \frac{z^2 + a^2 - r^2}{2ar} \end{aligned} \right\} \quad (26)$$

Going over to the limit for $r = 0$ as in the case of the ring-source, we find for the z -component of velocity along the z -axis,

$$(v_z)_{r=0} = -6\pi^2 a^3 \kappa \cdot \frac{4z^2 - a^2}{[z^2 + a^2]^{\frac{3}{2}}}.$$

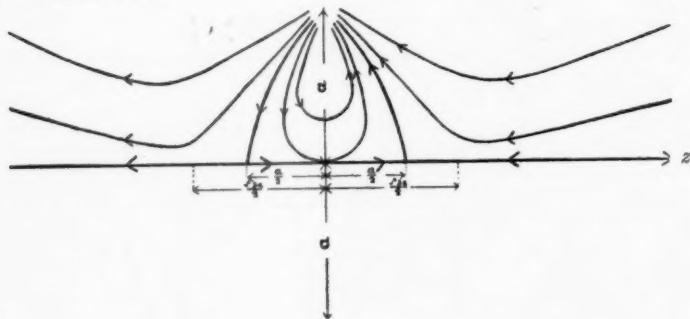
$$\text{For } z = 0, (v_z)_{r=0} = \frac{6\pi^2 \kappa}{a^2}.$$

$$\text{For } 0 < |z| < \frac{a}{2}, (v_z)_{r=0} > 0 \quad (\kappa > 0).$$

$$\text{For } \frac{a}{2} < |z| < \infty, (v_z)_{r=0} < 0.$$

The maximum values are at $z = 0$ when $(v_z)_{r=0} = \frac{6\pi^2 \kappa}{a^2}$, and at $z = \pm \frac{a\sqrt{3}}{2}$, where $(v_z)_{r=0} = -\frac{6\pi^2 \kappa}{a^2} \cdot \frac{2^8}{71}$.

From this we see that the stream-lines will approximately be as shown in the figure below:



Superposing upon this the motion due to a uniform stream parallel to the z -axis, the resultant velocity-potential will be given by

$$\bar{\phi}'_7 = -v_0 z + \frac{\pi\kappa}{r} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} \cdot \frac{kz}{a} \left\{ 2K - \frac{1+k^2}{k^2} E \right\}.$$

From the above figure and a uniform stream we can get an approximate idea of the resulting stream-lines. If we take $\kappa > 0$ and $v_0 > \frac{6\pi^2\kappa}{a^2} \cdot \frac{2^8}{71}$, then we shall get a body such as shown on Plate XVII, fig. 1; and if we take $\kappa < 0$, and $v_0 > \frac{6\pi^2\kappa}{a^2}$, then we get a body such as shown in Fig. 2.

Within such bodies, then, the stress-components will be given by

$$\hat{\theta}z = r \frac{\partial \bar{\phi}'_7}{\partial z} = -v_0 r + \frac{\pi\kappa}{r} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} k (B_1 K + B_2 E)$$

$$r\hat{\theta} = r \cdot \frac{\partial \bar{\phi}'_7}{\partial r} = \frac{\pi\kappa}{2ar} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} kz (C_1 K + C_2 E),$$

where the values of B_1, B_2, C_1, C_2 are given by equations 23 and 24.

The displacement u_θ will be given by

$$u_\theta = \frac{rz}{\mu} \left\{ -v_0 + \frac{\pi\kappa}{ar} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} k \left[2K - \frac{1+k^2}{k^2} E \right] \right\},$$

where the plane $z = 0$ is taken as the plane of zero displacement.

If r_1 is the radius of the body at the two distant ends, then the moment of the couples is $\frac{\pi v_0 r_1^4}{2}$.

SECTION 12.

Infinite Plate Source with Circular Hole.

To find the velocity-potential due to a uniform distribution of sources over an infinite plate with a circular hole in it, we shall firstly find that due to a distribution of sources over an infinite plate, and then that due to a distribution of sinks of the same strength over a finite circular plate. The sum of these two potentials shall give us the velocity-potential of a uniform distribution of sources over an infinite plate with a circular hole in it.

1. *Infinite Plate.*

Let the plate pass through the origin, and let the z -axis be perpendicular to it. It is evident that if in this case we take the potential due to the plate source to vanish at infinity, then at finite distances the potential will be infinite. We shall, therefore, in this case take the potential to be zero in the plane of the plate.

We have found that the velocity due to a circular ring-source at a point on its axis is given by

$$v = 6\pi^2 a^3 \frac{z}{(z^2 + a^2)^{\frac{3}{2}}} \quad (c > 0)$$

where a is the radius of the ring.

The velocity due to an infinite plate-source will therefore be

$$\begin{aligned} v &= 6\pi^2 c_1 z \int_0^{\infty} \frac{a^3 da}{(z^2 + a^2)^{\frac{3}{2}}} \\ &= 4\pi^2 c_1, \end{aligned}$$

where c_1 is the strength of the distribution.

We therefore have:

$$\text{For } z > 0, v = +4\pi^2 c_1 = -\frac{\partial \phi'_s}{\partial z}.$$

$$\text{For } z < 0, v = -4\pi^2 c_1 = -\frac{\partial \phi'_s}{\partial z},$$

where ϕ'_s is the velocity-potential of the motion.

Further,

$$\frac{\partial \phi'_s}{\partial r} = 0.$$

Hence for

$$\begin{aligned} z > 0, \phi'_s &= -4\pi^2 c_1 z \\ z < 0, \phi'_s &= +4\pi^2 c_1 z \end{aligned} \quad (27)$$

2. Circular Plate Sink.

From 22 we have that the velocity-potential of a ring sink of radius a is given by

$$\phi'_s = -\frac{4\pi c}{k} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} \{(2 - k^2)K - 2E\},$$

where the modulus k is given by

$$k^2 = \frac{4ar}{z^2 + (a + r)^2}.$$

The velocity-potential due to a distribution of sinks over a circular plate of radius a_1 will therefore be given by

$$\phi'' = -4\pi c_1 \int_0^{a_1} \frac{1}{k} \cdot \left(\frac{a}{r}\right)^{\frac{3}{2}} \{(2 - k^2)K - 2E\} da.$$

Now, the modulus, k , is less than one for all values of z and r , excepting $z = 0, r = a$, where $k^2 = 1$.

We may therefore write:

$$K = \frac{\pi}{2} \left\{ 1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 k^4 + \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 k^6 + \dots \right\}$$

$$E = \frac{\pi}{2} \left\{ 1 - \left(\frac{1}{2}\right)^2 k^2 - \left(\frac{1}{2 \cdot 4}\right)^2 3k^4 - \left(\frac{1 \cdot 3}{2 \cdot 4 \cdot 6}\right)^2 5k^6 - \dots \right\}.$$

These expansions are valid at all points excepting the ring of points $r = a$ in the plane $z = 0$.

In the neighbourhood of these points the series converge slowly. These expansions may also be written:

$$\begin{aligned} K &= \frac{\pi}{2} \left\{ 1 + \sum_{n=1}^{\infty} \left(\prod_{p=1}^n \left(\frac{2p-1}{2p} \right)^2 \right) k^{2n} \right\} \\ E &= \frac{\pi}{2} \left\{ 1 - \sum_{n=1}^{\infty} \left(\prod_{p=1}^{n-1} \left(\frac{2p-1}{2p} \right)^2 \cdot \frac{2n-1}{(2n)^2} \right) \cdot k^{2n} \right\} \\ \therefore (2-k^2)K - 2E &= \frac{\pi}{2} \left\{ 2 \sum_{n=1}^{\infty} \left(\prod_{p=1}^n \left(\frac{2p-1}{2p} \right)^2 \right) k^{2n} + 2 \sum_{n=1}^{\infty} \left(\prod_{p=1}^{n-1} \left(\frac{2p-1}{2p} \right)^2 \cdot \frac{2n-1}{(2n)^2} \right) k^{2n} \right. \\ &\quad \left. - \sum_{n=1}^{\infty} \left(\prod_{p=1}^{n-1} \left(\frac{2p-1}{2p} \right)^2 \right) k^{2n} \right\} \\ &= \frac{\pi}{2} \left\{ \sum_{n=1}^{\infty} \left[\frac{2(2n-1)^2}{4n^2} \prod_{p=1}^{n-1} \left(\frac{2p-1}{2p} \right)^2 + \frac{2(2n-1)}{4n^2} \prod_{p=1}^{n-1} \left(\frac{2p-1}{2p} \right)^2 \right. \right. \\ &\quad \left. \left. - \prod_{p=1}^{n-1} \left(\frac{2p-1}{2p} \right)^2 \right] k^{2n} \right\} \\ &= \frac{\pi}{2} \sum_{n=2}^{\infty} \frac{n-1}{n} \cdot \prod_{p=1}^{n-1} \left(\frac{2p-1}{2p} \right)^2 \cdot k^{2n} \quad (k^2 < 1) \end{aligned}$$

Substituting this value of $(2-k^2)K - 2E$ and $k^2 = \frac{4ar}{z^2 + (a+r)^2}$ in the integral for ϕ''_s , we get:

$$\phi''_s = -\pi^2 c_1 \int_0^{a_1} \sum_{n=2}^{\infty} \frac{2^{2n}}{r^{n-2}} \cdot \frac{n-1}{n} \cdot \prod_{p=1}^{n-1} \left(\frac{2p-1}{2p} \right)^2 \cdot \frac{a^{n+1} da}{[z^2 + (a+r)^2]^{\frac{2n-1}{2}}}$$

Since this series converges uniformly for all values of z and r excepting $z=0$, $r=a$, therefore we may integrate term by term, and hence we have:

$$\phi''_s = -\pi^2 c_1 \sum_{n=2}^{\infty} \frac{2^{2n}}{r^{n-2}} \cdot \frac{n-1}{n} \cdot \prod_{p=1}^{n-1} \left(\frac{2p-1}{2p} \right)^2 \int_0^{a_1} \frac{a^{n+1} da}{[z^2 + (a+r)^2]^{\frac{2n-1}{2}}} \quad (k^2 < 1)$$

The integral

$$\int_0^{a_1} \frac{a^{n+1} da}{[z^2 + (a+r)^2]^{\frac{2n-1}{2}}}$$

may immediately be solved by making the transformation $a+r = z \tan \xi$.

This reduces the integral to

$$\frac{1}{z^{2(n-1)}} \int_{\tan^{-1} \frac{r}{z}}^{\tan^{-1} \frac{r+a_1}{z}} (z \tan \xi - r)^{n+1} \cdot \cos^{2n-3} \xi \cdot d\xi.$$

It may also be solved by first of all reducing the integral to I_0 by means of the reduction formula

$$\frac{a_1^p}{[z^2 + (a_1 + r)^2]^{\frac{2n-3}{2}}} = (p-2n+3) I_{p+1} + r(2p-2n+3) I_p + p(z^2 + r^2) I_{p-1},$$

which is obtained by differentiating $[z^2 + (a+r)^2]^{\frac{2n-3}{2}}$ with respect to a , and integrating, and where

$$I_p = \int_a^{a_1} \frac{a^p \cdot da}{[z^2 + (a+r)^2]^{\frac{2n-1}{2}}}.$$

The integral I_0 may again be evaluated by applying the transformation

$$a + r = z \tan \xi.$$

We get

$$I_0 = \frac{1}{z^{2(n-1)}} \int_{\tan^{-1} \frac{r+a_1}{z}}^{\tan^{-1} \frac{r}{z}} (1-t^2)^{n-2} dt$$

where $t = \sin \xi$.

If now

$$\phi_8 = \phi'_8 + \phi''_8,$$

then ϕ_8 will be the velocity-potential due to a uniform distribution of sources over an infinite plate with a circular hole in it. The stream-lines of such a distribution will be approximately such as shown in Plate XVIII, fig. 1.

If we superpose upon this motion a uniform stream of velocity v_0 , whose velocity-potential ϕ_0 is from 14 given by

$$\phi_0 = -v_0 z,$$

then the stream-lines of the motion, whose velocity-potential is given by

$$\phi_8 = \phi_0 + \phi'_8$$

will approximately be as shown in Plate XVIII, fig. 2.

From this, then, we get a body such as shown in Plate XVIII, fig. 3.

Within such bodies, then, the stress-components will be given by

$$\widehat{r\theta} = r \cdot \frac{\partial \phi_8}{\partial r}$$

$$\widehat{\theta z} = r \cdot \frac{\partial \phi_8}{\partial z}$$

and the displacement u_θ by

$$u_\theta = \frac{r}{\mu} \left\{ \phi_8 + c \right\}$$

the constant c determining the surface of zero displacement.

$$\text{At } z = -\alpha, \quad v_{-\alpha} = v_0 - 4\pi^2 c_1.$$

$$\text{At } z = +\alpha, \quad v_{+\alpha} = v_0 + 4\pi^2 c_1.$$

The moment of the terminal couples will therefore be given by

$$M = - \int_0^{r_1} \int_0^{2\pi} r^3 (\widehat{\theta z})_{z=-\alpha} dr \cdot d\theta = - \int_0^{r_1} \int_0^{2\pi} r^3 (\widehat{\theta z})_{z=+\alpha} dr \cdot d\theta.$$

$$\begin{aligned}
 &= (v_0 - 4\pi^2 c_1) \int_0^{r_1} \int_0^{2\pi} r^3 \cdot dr \cdot d\theta = (v_0 + 4\pi^2 c_1) \int_0^{r_2} \int_0^{2\pi} r^3 dr \cdot d\theta \\
 &= \frac{\pi(v_0 - 4\pi^2 c_1) r_1^4}{2} = \frac{\pi(v_1 + 4\pi^2 c_1) r_2^4}{2},
 \end{aligned}$$

where r_1 and r_2 are the radii of the body at $z = -\alpha$ and α respectively.

From this result we further note that

$$\frac{r_2^4}{r_1^4} = \frac{v_0 - 4\pi^2 c_1}{v_0 + 4\pi^2 c_1}.$$

If another infinite plate source or sink with a circular hole in it, its plane passing through another point on the z -axis, be superposed upon the motion whose velocity-potential is given by ϕ_0 , then some other very interesting and practical cases can be obtained, and the problem seems to become ever more fascinating.

APPENDIX.

Volumes and Surfaces of n -dimensional Spheres.

The volume of an n -dimensional sphere of radius r will be given by

$$V_n = \int \int \dots \int dx_1 dx_2 \dots dx_n$$

integrated over the region

$$x_1^2 + x_2^2 + \dots + x_n^2 \leq r^2.$$

We now make the n -dimensional polar transformation,

$$\begin{aligned}
 x_1 &= r \cos a_{n-1} \\
 x_2 &= r \sin a_{n-1} \cos a_{n-2} \\
 x_3 &= r \sin a_{n-1} \sin a_{n-2} \cos a_{n-3} \\
 &\vdots \\
 x_{n-1} &= r \sin a_{n-1} \sin a_{n-2} \dots \sin a_2 \cos a_1 \\
 x_n &= r \sin a_{n-1} \sin a_{n-2} \dots \sin a_2 \sin a_1
 \end{aligned}$$

If J_n is the Jacobian of the transformation, then

$$V_n = \int_{r=0}^r \int_{a_1=0}^{\pi} \dots \int_{a_{n-1}=0}^{\pi} \int_{a_n=0}^{2\pi} \bar{V}_n(r, a_1, \dots, a_{n-1}) dr \cdot da_{n-1} \dots da_1.$$

But

$$J_n = r^{n-1} \prod_{p=1}^{n-2} \sin^p a_{p+1}.$$

$$\therefore V_n = \frac{2\pi r^n}{n} \prod_{p=1}^{n-2} \int_{a=0}^{\pi} \sin^p a_{p+1} da_{p+1}.$$

$$\text{If } I = \int_{\substack{a=0 \\ p+1}}^{\pi} \sin_p a_{p+1} da_{p+1},$$

then we get from the reduction formula

$$I_p = \frac{p-1}{p} I_{p-2}.$$

$$\begin{aligned} I_p &= \frac{(p-1)(p-3) \dots 2}{p(p-2) \dots 3} \cdot 2 = \frac{2^p \left[\left(\frac{p-1}{2} \right)! \right]^2}{p!} \text{ if } p \text{ is odd.} \\ &= \frac{(p-1)(p-3) \dots 1}{p(p-2) \dots 2} \cdot \pi = \frac{p!}{2^p \left[\left(\frac{p}{2} \right)! \right]^2} \cdot \pi \text{ if } p \text{ is even.} \end{aligned}$$

Or we may write:

$$\begin{aligned} I_p &= I_{2s+1} = \frac{2^p \left[\left(\frac{p-1}{2} \right)! \right]^2}{p!} = \frac{2^{2s+1} [(2s)!]^2}{(2s+1)!} \\ I_p &= I_{2s} = \frac{p!}{2^p \left[\left(\frac{p}{2} \right)! \right]^2} = \frac{(2s)!}{2^{2s} [s!]^2} \cdot \pi. \end{aligned}$$

Hence, if n is odd ($n = 2m+1$).

$$\begin{aligned} V_n &= V_{2m+1} = \frac{2\pi^m \cdot r^{2m+1}}{2m+1} \prod_{s=0}^{m-1} \frac{2^{2s+1} (s!)^2}{(2s+1)!} \prod_{s=1}^{m-1} \frac{(2s)!}{2^{2s} (s!)^2} \\ &= \frac{2^{m+1} \cdot \pi^m \cdot r^{2m+1}}{2m+1} \prod_{s=0}^{m-1} \frac{1}{2s+1} \longrightarrow (1) \end{aligned}$$

Again, if n is even ($n = 2m$)

$$\begin{aligned} V_n &= V_{2m} = \frac{\pi^m \cdot r^{2m}}{m} \prod_{s=0}^{m-2} \frac{2^{2s+1} (s!)^2}{(2s+1)!} \prod_{s=1}^{m-1} \frac{(2s)!}{2^{2s} (s!)^2} \\ &= \frac{[2(m-1)!]}{2^{m-1} \cdot m [(m-1)!]^2} \cdot \pi^m \cdot r^{2m} \prod_{s=0}^{m-2} \frac{1}{2s+1} \longrightarrow (2) \end{aligned}$$

Again we note that the area of a circle in space of n -dimensions is equal to the volume of a sphere in $(n-1)$ -dimensions, and the circumference of the circle in n -dimensions is equal to the surface of a sphere in $(n-1)$ -dimensions.

Of, therefore,

V_n = volume of sphere in n -dimensions,

S_n = surface of sphere in n -dimensions,

A_n = area of circle in n -dimensions.

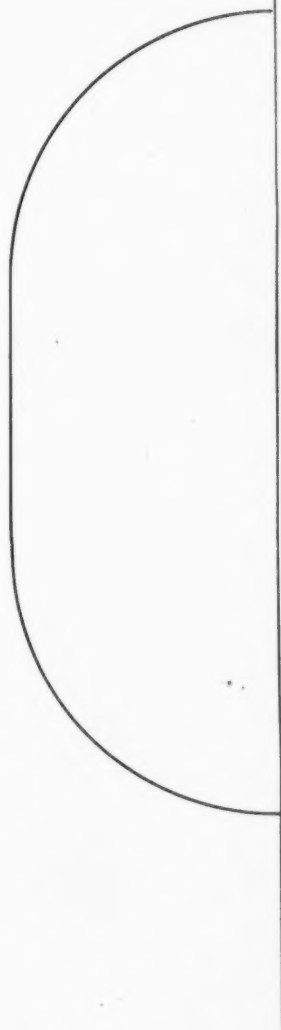
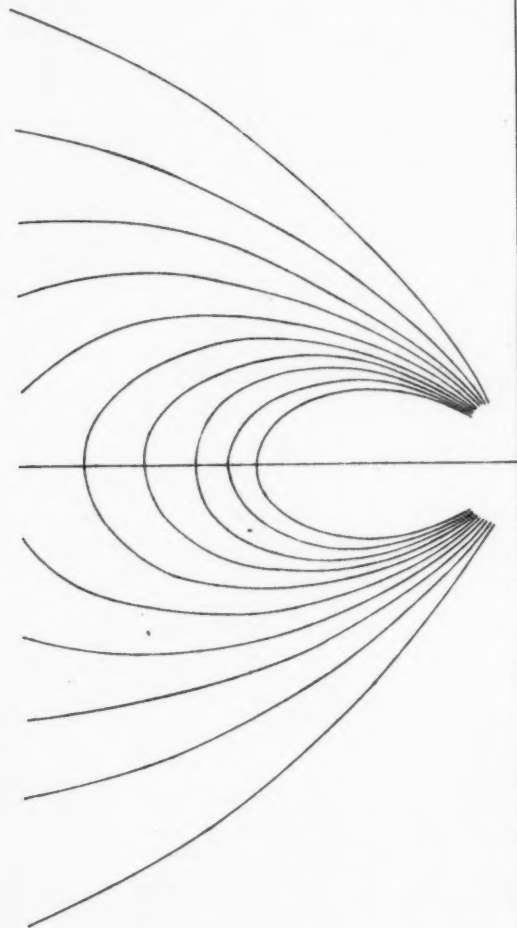
C_n = circumference of circle in n -dimensions,

Then we have

$$\left. \begin{aligned} S_n &= \frac{dV_n}{dv} \\ A^n &= V^{n-1} \\ C_n &= \frac{dA^n}{dr} = \frac{dV_{n-1}}{dr} = S_{n-1} \end{aligned} \right\} \quad (3)$$

From 1, 2, and 3 we have

$$\begin{aligned} A_4 &= V_3 = \frac{4}{3}\pi r^3; & C_4 &= S_3 = 4\pi r^2 \\ A_5 &= V_4 = \frac{\pi^2 r^4}{2}; & C_5 &= S_4 = 2\pi^2 \cdot r^3 \\ A_6 &= V_5 = \frac{8\pi^2 r^5}{15}; & C_6 &= S_5 = \frac{8\pi^2 r^4}{3} \\ A_7 &= V_6 = \frac{\pi^3 r^6}{6}; & C_7 &= S_6 = \pi^3 r^5. \\ - & - & - & - & - & - \end{aligned}$$



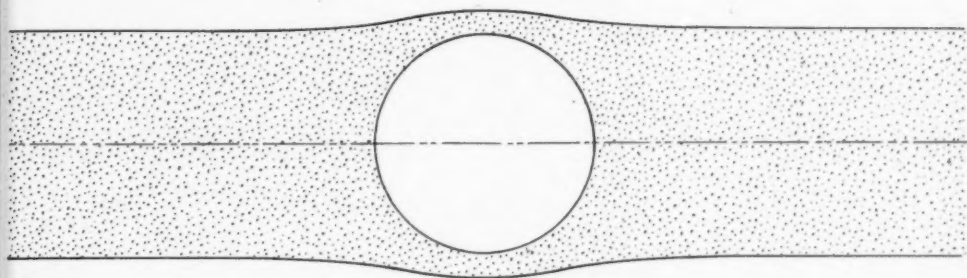
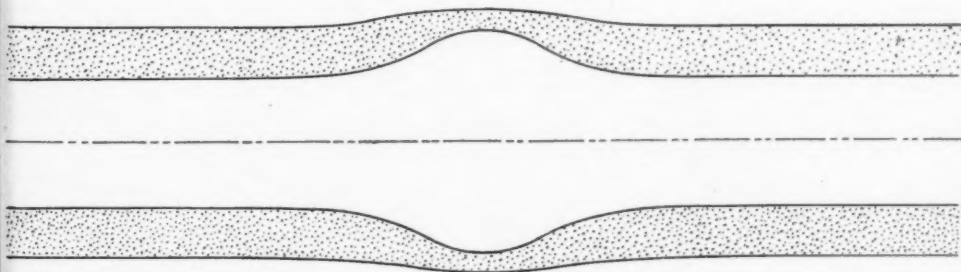


Plate XI.



Scale 15 cms. = 1 unit.

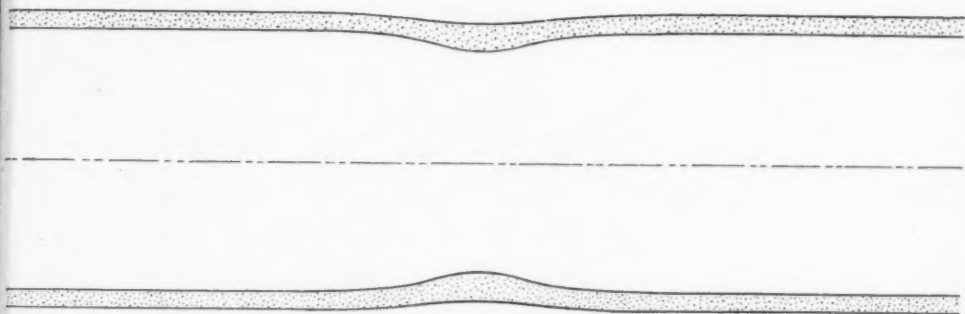
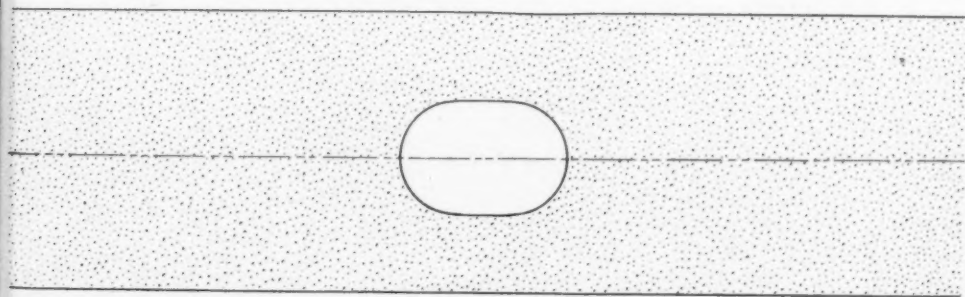
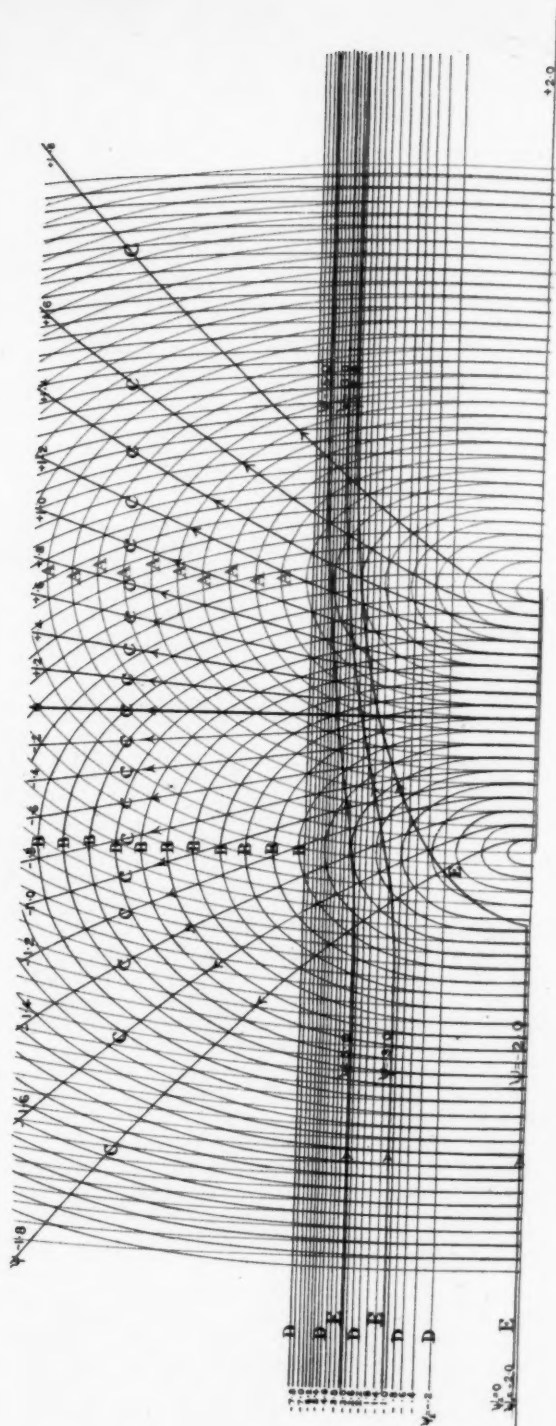


Plate XIII.

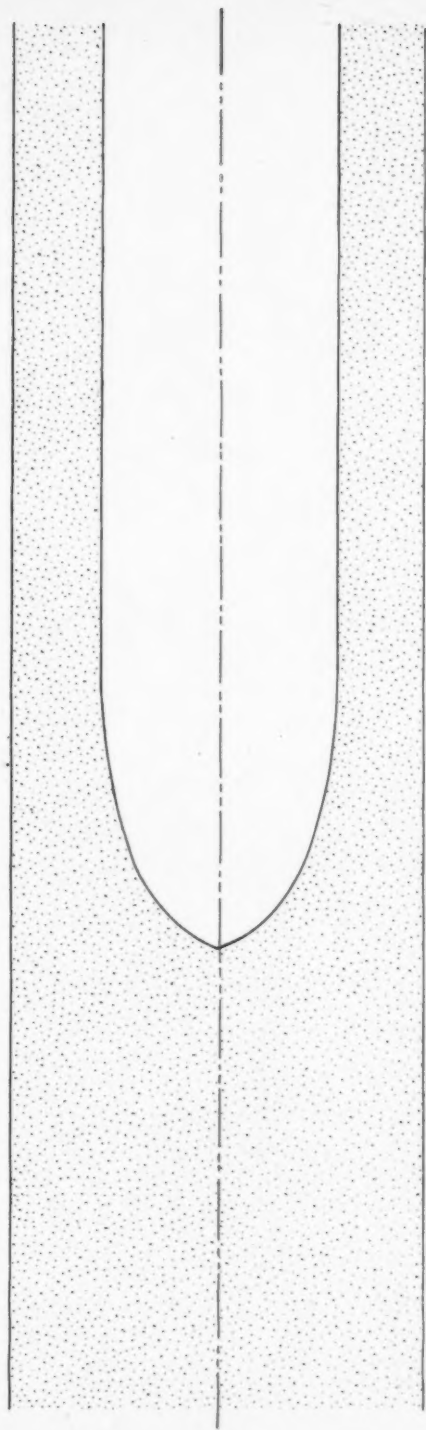


Scale 15 cms. = 1 unit



$$\begin{aligned}
 A \quad \psi' &= -\frac{2(3-\frac{1}{2})^2 + \gamma^2}{\{(3-\frac{1}{2})^2 + \gamma^2\}^{\frac{1}{2}}} \\
 B: \quad \psi' &= +\frac{2(3+\frac{1}{2})^2 + \gamma^2}{\{(3+\frac{1}{2})^2 + \gamma^2\}^{\frac{1}{2}}} \\
 C: \quad \psi' &= \psi'_1 + \psi'_2 = -\frac{2(3-\frac{1}{2})^2 + \gamma^2}{\{(3-\frac{1}{2})^2 + \gamma^2\}^{\frac{1}{2}}} + \frac{2(3+\frac{1}{2})^2 + \gamma^2}{\{(3+\frac{1}{2})^2 + \gamma^2\}^{\frac{1}{2}}} \\
 D: \quad \psi_2 &= -16\gamma^4 \\
 E: \quad \psi &= \psi_1 + \psi_2 = -16\gamma^4 - \frac{2(3-\frac{1}{2})^2 + \gamma^2}{\{(3-\frac{1}{2})^2 + \gamma^2\}^{\frac{1}{2}}} + \frac{2(3+\frac{1}{2})^2 + \gamma^2}{\{(3+\frac{1}{2})^2 + \gamma^2\}^{\frac{1}{2}}}
 \end{aligned}$$

Scale 25 cms. = 1 unit



Scale 10 cms. = 1 unit.

Fig 1.

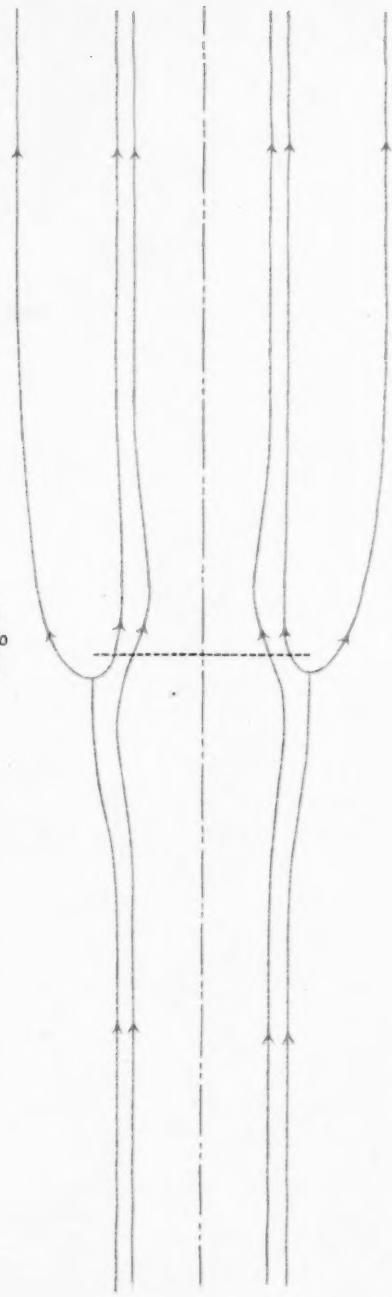


Fig 2.

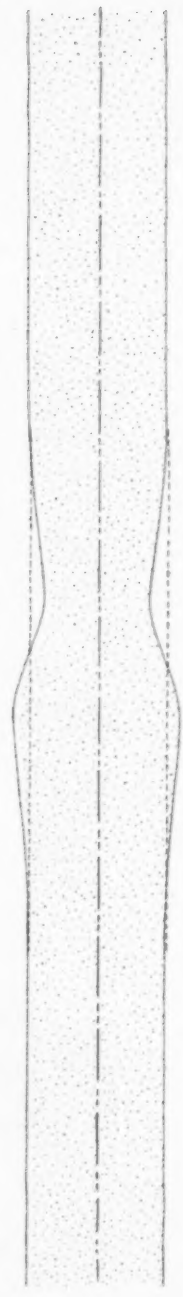


Fig 1.

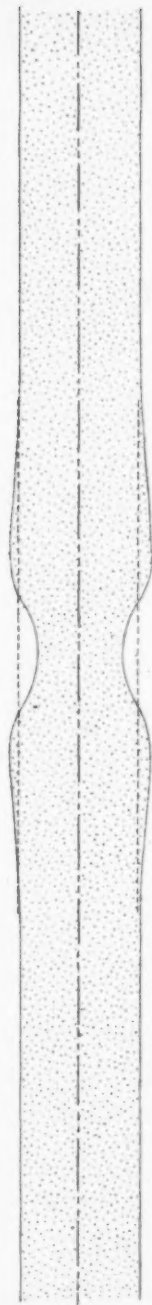


Fig 2.

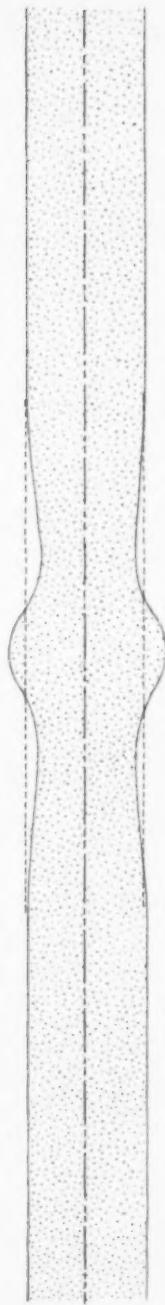


Fig 1.

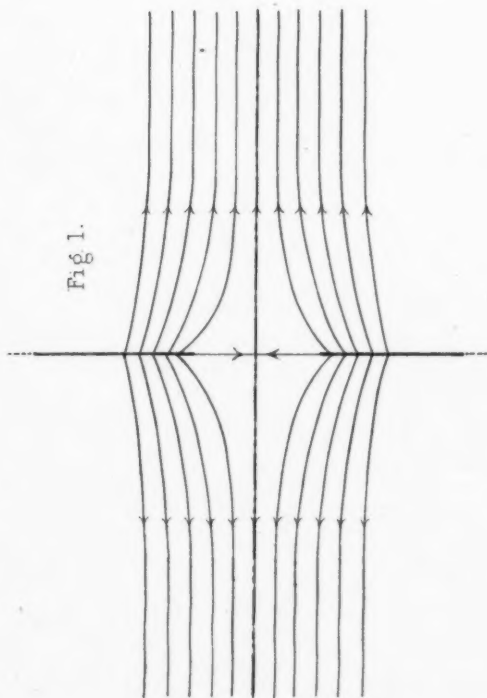


Fig 2.

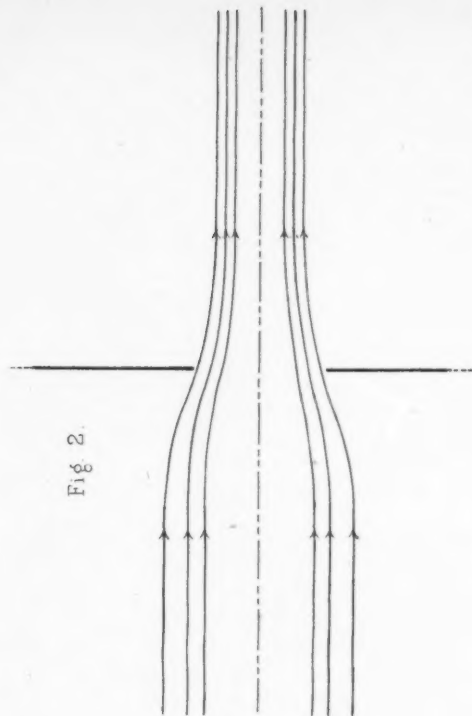
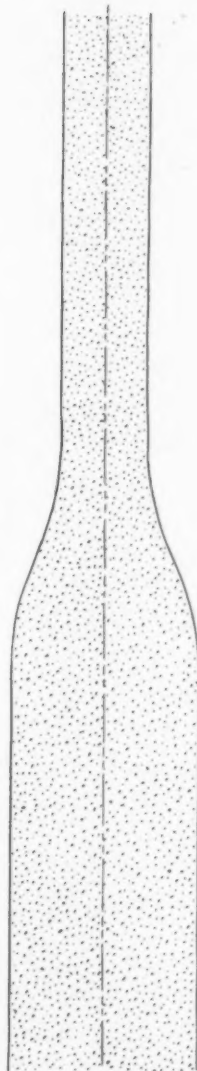


Fig 3.



NOTE ON THE STRUCTURE OF THE GENITAL ORGANS OF A
TRUE HERMAPHRODITE.

By H. V. EXNER, M.A.

(From the Physiology Laboratory, South African College.)

The true nature of this hermaphrodite, whose tissues I obtained from the Anatomy Department of the South African College, was only recognised after a thorough and minute examination of its genital organs. These belonged to a human being of the Bantu race, whose age at the time of death was about twenty years. During life this person passed as a male, was of unsound mind, and had the general appearance of a Kaffir girl with fairly large breasts and female pelvis.

A short, penis-like structure was found; labia majora covered with sparse hairs, but no external genital orifice beyond the opening of the urethra on tip of the glans penis. Internally a small, isolated vagina closed at both ends was found. A uterus, the mere remnant of the fundus and having no connection with vagina, had attached on either side a well-developed Fallopian tube, broad ligament, and round ligaments. Running through the uterus from the attachment of one Fallopian tube to the other was a minute lumen. The left ovary was of normal size, while the right ovary was double the size of the left due to acute inflammation of that gland.

A histological examination of all these structures revealed the following:

Penis.—The normal structure of a penis is present, only smaller in size and with a small urethra passing along its inferior surface and opening by a vertical slit at the tip of the glans penis. A cross-section of this organ shows the corpus cavernosum and urethra as in a normal penis. Another section taken just below the bladder shows a very close resemblance to prostatic tissue. At a lower level a cross-section shows bulbous tissue and corpus cavernosum.

Uterus.—A cross-section of the small atrophied uterus shows a small lumen lined with mucous membrane. The openings of the glands in this mucous membrane are lined with columnar epithelial tissue. The walls of the uterus consist of plain muscle, rich in blood-supply. The lumen of the uterus would just about admit an ordinary pin.

Left Ovary.—The main mass of the gland consists of fibrous stroma not unlike that of a normal ovary. It has a very rich blood-supply and contains a few peculiar glandular structures. Small structures, in appearance like Graafian follicles containing unripe ova, are found, but less numerous than in a normal human ovary. The ova have a mean diameter of $\cdot 033$ mm., with nuclei of diameter $\cdot 013$ mm. The sizes of these ova correspond closely to those of rabbit ova examined.

Right Ovary.—In parts this organ shows the same structure as the left ovary—i. e. fibrous stroma, peculiar glandular structures, Graafian follicles, and unripe ova. Other parts of the organ, however, are quite distinct and have the appearance of testicular substance. Here we find tubules very similar to seminiferous tubules with a lumen in most; no actual spermatozoa, however, could be detected, leading to the assumption that this testicular tissue could not have been functional. In parts this substance had the tubular appearance of epididymis, showing in some sections a distinct tubular outlet lined with columnar epithelium similar to the ductus deferens.

COLOUR AND CHEMICAL CONSTITUTION.

PART IV.—THE REMAINING PHTHALEINS.

BY JAMES MOIR, M.A., D.Sc., F.I.C.

In continuation of the systematic examination of the absorption-spectra of the phthaleins described in the three earlier parts of this work (1917), I have now prepared the phthaleins of all the remaining phenolic substances which are obtainable or capable of being made in South Africa. The following are the data observed in these new substances :

A. DIRECT SUBSTITUTION-PRODUCTS OF ORDINARY PHENOL-PHTHALEIN.

(1) *a-anthrol-phthalein*, from *a-anthrol* (Journ. Chem. Soc. Lond., 1916, p. 774) and phthalic anhydride, gives a brownish-green solution in alkali possessing an olive-green fluorescence. Its absorption-band is very near the red end, and can only be seen by use of direct sunlight, the centre being at about λ 740. In conc. H_2SO_4 the band is broader and has its centre about λ 720.

(2) *Phenol-a-anthrol-phthalein*, from anthrol and oxybenzoylbenzoic acid, gives a greenish-blue colour in alkali. The centre of the absorption-band is at λ 628. This substance is the (mono) xylylidene* derivative of phenol-phthalein, and is interesting as showing the greatest displacement of the absorption-band observed (a diminution of frequency of 13 per cent. for the single C_6H_5 group which constitutes the difference between anthracene and benzene). In the case of phenol-*a-naphthol*phthalein the fall of frequency is 9.5 per cent. for the single butylidene group, which constitutes the difference between naphthalene and benzene. In conc. H_2SO_4 the above anthrol compound is purplish blue, with its band at λ 611.

(3) *Phenol metacresolphthalein* or *metamethyl-phenolphthalein* is purplish-pink in alkali, the band being at λ 569. In H_2SO_4 the colour is salmon and the band at λ 526. In connection with metacresolphthalein, described in Part I, I have now to issue a correction. The centre of its absorption-band as it λ 584, not λ 590. The specimen examined last year contained a trace of thymolphthalein, which caused a broadening of its band and a mis-estimate of its centre.

* Xylylidene is naphthalene minus $\text{CH}\cdot\text{CH}$.

(4) *Phenolphthalein dimethyl ether*.—This long-known substance, which gives no colour in alkali, gives a salmon colour in H_2SO_4 with absorption-band centre at λ 508. Phenolphthalein monomethyl ether, examined in the same way, showed its band-centre to be at λ 503: that of phenolphthalein itself in H_2SO_4 is at λ 499.

(5) *Phenolphthalein monosuccinic acid*, made by condensing oxybenzoylbenzoic acid with *o*-oxyphenylsuccinic acid (itself made from coumarine), is purplish-pink in alkali with band-centre at λ 564. In H_2SO_4 it is salmon-coloured with band-centre at λ 502.

(6) *Phenolphthalein-disuccinic acid*, from the above acid with phthalic anhydride, could only be obtained in traces. Its band-centre in alkali is at λ 575, the displacement of the band from that of phenolphthalein being about twice that of the foregoing monosuccinic acid.

(7) *Thymolmetacresolphthalein*.—This substance, which is intermediate in constitution between *m*-cresol- and thymol-phthaleins, and thus enabled the influence of the *o*-isopropyl group to be studied, was made by condensing metacresol with a new acid, *thymoylbenzoic acid* (2-methyl-5-isopropyl-4-oxybenzoylbenzoic acid). The latter is made from thymolphthalein in the same way as oxybenzoylbenzoic acid is made from phenolphthalein (see Part II). This new phthalein is violet in alkali with its band-centre at λ 590 almost exactly across the D line of the solar spectrum. In H_2SO_4 it is pink with a broad band, the centre of which is at about λ 530.

(8) *Thymol- α -naphtholphthalein*, from α -naphthol and thymoylbenzoic acid, is bluish-green in alkali, the centre of its absorption-band being at λ 633. This forms an interesting example of the fact that the effects of the substitutions are additive, *i.e.* that the colour and position of the absorption-band of very complex phthaleins can be predicted from data obtained from simpler phthaleins. Thymolnaphtholphthalein can be looked on either as (*a*) thymolphenolphthalein, to which the butylidene group has been added (converting phenol into naphthol), or (*b*) as phenolnaphtholphthalein, to which the methyl and isopropyl groups have been added (converting phenol—in the other ring—into thymol). Now, in case (*a*) the value of the butylidene group can be ascertained by comparing the spectrum of phenol- α -naphtholphthalein with that of phenolphthalein. The central wave-lengths of these substances are 607 and 554, so that the single butylidene group which makes the difference between them has raised the wave-length of the band by 9.5 per cent. Applying, therefore, a correction of +9.5 per cent. to the figure for phenolthymolphthalein (previously ascertained to be λ 578, see Part II), we get $1.095 \times 578 = \lambda$ 633 as the calculated value for thymol- α -naphtholphthalein. In case (*b*), in the same way, we take the experimentally ascertained value for phenolnaphtholphthalein (*viz.* λ 607) and multiply it by the ratio indicating the conversion of one phenol group into a thymol group (*viz.* 1.044 or $578 \div 554$), this

giving a second theoretical value for the band-centre in the supposed unknown thymolnaphtholphthalein, viz. 607×1.044 or $\lambda 634$.

The observed value being $\lambda 633$, it is seen that both predictions are accurate.


(9) *Thymol- α -anthrophthalein*, from thymoylbenzoic acid and α -anthrol, is bluish-olive in alkali with an indefinite spectrum consisting of a strong dulling of the yellowish-green and the outer red. The centre of the latter band is probably $\lambda 710$, and the other may be due to the presence of a little thymolphthalein. In H_2SO_4 the colour is emerald and the absorption-band sharp with centre at $\lambda 636$.


(10) *Monophenyl ether of phenolphthalein*.—An attempt to make this substance from oxybenzoylbenzoic acid and $(C_6H_5)_2O$ using H_2SO_4 as condensing agent gave a product which was mostly composed of phenolphthalein, but also contained another substance having a broad band with centre at $\lambda 470$. It is quite probable that the latter substance is merely β -oxyanthraquinone (see Part III), the absorption-band of which was observed to be at $\lambda 475$ (α -oxyanthraquinone has its band-centre at $\lambda 490$).

Secondary bands of the phthaleins.—The existence of an absorption-band possessing $\frac{2}{3}$ of the wave-length and $\frac{2}{3}$ of the frequency of the visible band having been suspected (and indicated in Part I), an examination of the extreme violet end was attempted. Lack of quartz apparatus prevented the application of the photographic method. It was found that strong solution of alkaline phenolphthalein absorbs the extreme violet, the edge of the band coming up to $\lambda 380$, which is consistent with the presence of a band with its centre at $\frac{2}{3} \times 554$ or $\lambda 369$. In the case of thymolphthalein in strong solution in alkali the absorption of the violet is almost total, the edge of the absorption extending to $\lambda 420$ ($\frac{2}{3} \times 597 = \lambda 398$ calculated for centre).

B. DERIVATIVES OF *o-p*-PHENOLPHTHALEIN.

(a) *Phenolquinolphthalein*.—This is obtained as usual from oxybenzoylbenzoic acid and quinol, but purification of the product was unsuccessful, and since the absorption-band is vague, no further investigation was made. The colour of the phthalein in alkali is maroon: band-centre about $\lambda 540$, i.e. lower than that of any of the para-phthaleins. In H_2SO_4 the phthalein gives a yellow solution, the band being extremely far down in the blue, at about $\lambda 435$ —a position similar to that of fluoresceine in H_2SO_4 .

(b) *Ethyl ether of foregoing phthalein*.—This was obtained by using EtO  OH (from phenetidine) instead of quinol. The product was also unsatisfactory. The alkaline colour is dirty purple with a very broad band at about $\lambda 552$ and signs of another at about $\lambda 490$.

(c) *Thymol-p-ethoxyphenol-phthalein* from EtO  OH and thymoyl-

benzoic acid is blue in alkali with central wave-length at λ 598. It differs from thymolphthalein in giving an orange solution in H_2SO_4 in which there is no band at λ 550, the absorption being in the violet.

(d) *Thymol-p-cresol-phthalein* also resembles thymolphthalein in alkaline solution, its central wave-length being at about λ 595; in H_2SO_4 it is also orange (thymolphthalein purplish-pink) with only violet-absorption. The additive property may again be illustrated here:

$$\frac{\text{phenolparacresolphthalein}}{\text{phenolphthalein}} = \frac{572}{554}, \text{ and } \frac{\text{phenolthymolphthalein}}{\text{phenolphthalein}} = \frac{578}{554}$$

$$\therefore \text{calculated value for thymol-p-cresol-phthalein is } \frac{572 \times 578}{554} = 597.$$

The agreement is better if the *differences* are merely added together: $572 + 578 - 554 = 596$.

(e) *Thymol-β-naphtholphthalein* is also indefinite: it is bluish-green in alkali with absorption-centre about λ 700, and like the rest of the class is orange in H_2SO_4 with violet absorption and no fluorescence.

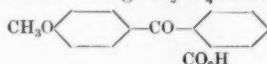
(f) *Phenol-β-anthrophthalein* is olive-green in alkali with a band at the treme red: its centre is about λ 730.

C. DERIVATIVES OF PHENYLPHENOLPHTHALEIN (OXYDIPHENYLPHTHALIDE).

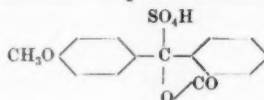
Phenylmetacresolphthalein.—Condensation of benzoylbenzoic acid with metacresol yielded a phthalein which, although giving a salmon colour in H_2SO_4 (central-wave-length λ 506), gave a perfectly colourless solution in alkali whatever strength of the latter was used. To explain this is difficult: one can only assume that since oxydiphenylphthalide itself is easily bleached by excess alkali, the methyl group in the new substance has so exaggerated this property that the quinonoid phase only exists momentarily, passing immediately into the colourless carbinol phase.

Phenylparacresolphthalein and *phenylparaoxybenzoic-acid-phthalein* were both yellowish-orange in alkali. The former had no band and the latter a faint, broad one at about λ 486.

The absorption-band of the orange H_2SO_4 solution of *anisoylbenzoic acid*



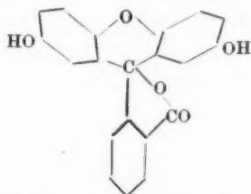
was observed to be at λ 466 about 7 units higher than that of the unmethylated substance (Part II). These almost unique sharp bands in the blue are due to the formation of phthalein-like sulphates, thus—



Similarly, the absorption-band of xanthone, the simplest fluorescent substance, in H_2SO_4 was found to be at λ 410.

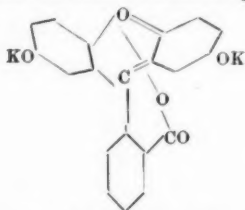
Note on the guaiacol-phthaleins.—It having been recently shown that when guaiacol is brominated the bromine does not enter *para* to the $-\text{OH}$ but is *para* to the $-\text{OCH}_3$ group (Journ. Chem. Soc., Lond., 1917, p. 941), it becomes probable that guaiacolphthalein and phenolguaiacolphthalein also have the methoxyl group *para* to the central-linking carbon, and consequently have the free $-\text{OH}$ group of guaiacol *meta* to this carbon. Experiment confirmed this, for when 5-bromoguaiacol was condensed with oxybenzoylbenzoic acid and H_2SO_4 at a low temperature, the phthalein produced was phenolguaiacolphthalein, bromine having been eliminated. This could scarcely be imagined unless the linkage to a phthalein occurred at the same point as the bromine came off at. The dibromoguaiacolphthalein described in Part I, obtained by direct bromination, has, therefore, probably one bromine in each ring *ortho* to the $-\text{OH}$ groups and also *ortho* to the central carbon.

Note on quinolphthalein.—The ordinary formulation of this substance:

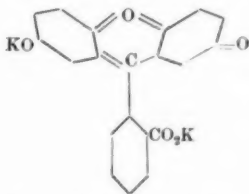


is not quinonoid and does not account for its colour.

The present writer now suggests two (tautomeric) formulae for the salts of this substance, which are quinonoid.



and



These two formulae are almost identical when it is considered that the K atoms are ionised off the molecule.

Some General Quantitative Conclusions on the Effect of Substitutions on Phenolphthalein.

One <i>o</i> -methyl	group raises the absorption wave-length by 1.5 per cent.				
Two "	groups raise	"	"	2.9	"
One <i>m</i> -methyl	group raises	"	"	2.7	"
Two "	groups raise	"	"	5.5	"
One <i>o</i> -isopropyl	group raises	"	"	1.3	" (?)
Two "	groups raise	"	"	2.4	"
One <i>m</i> -isopropyl	group raises	"	"	3.0	"
Two "	groups raise	"	"	5.6	"
Replacement of phthalic CO by					
SO ₂	raises	"	"	1.4	"
Two <i>o</i> -Br atoms	raise	"	"	1.7	"
Four " " "		"	"	5.0	"
Four <i>o</i> -I " "		"	"	6.0	"
The oxygen "oxo"-linkage in					
fluorescein depresses		"	"	10.0	"

There are many other additive relationships which are, however, less regular than these, possibly owing to difficulties of observation only.

ON THE GENERA *DIPLOCYSTIS* AND *BROOMEIA*.

BY I. B. POLE EVANS, M.A., D.Sc., F.L.S., and AVERIL M.
BOTTOMLEY, B.A.

(With Plates XIX-XXIII.)

The genus *Diplocystis* was founded by Berkeley and Curtis in 1867 to describe a peculiar gasteromycetous fungus collected in the island of Cuba by Charles Wright.

Hitherto it has been regarded as monotypic and has only been recorded from Cuba, Bahamas, and the West Indies. The fungus is known as *Diplocystis Wrightii*, Berk. and Curt.

Some beautiful specimens of *Diplocystis* have recently been sent to us from Portuguese East Africa, and as it is the first recorded occurrence of this interesting genus from Africa, it is made the subject of the present note.

A single specimen was, in the first instance, forwarded for identification by Mr. T. R. Sim, of Pietermaritzburg, who had received it from the Rev. H. A. Junod, of Lourenço Marques. The latter gentleman very shortly afterwards forwarded further specimens to Pretoria, and stated that—"The natives call it 'Fole da mangapfi'—viz. the tobacco of the hawk—because, they say, it appears at the same time as the hawk, which is said to remain hidden during the winter to appear at the rainy season." Mr. Junod also remarked that "it is found generally in old fields and gardens not cultivated the preceding year."

Our examination of these specimens has revealed the fact that the African material is not identical with that from Cuba. We propose, therefore, to describe it as *D. Junodii*, nov. spec.

***Diplocystis Junodii*, Pole Evans and Bottomley, nov. spec.**

Dense gregaria; massa fungina formae irregularis, 2-6 cm. lata \times 2-7 cm. longa, usque 8-60 individua amplexens; exoperidio pateriformi, margine definito erecto et leniter incurvato; endoperidio subgloboso, sessili, scabro et subtiliter floccoso, griseo vel umbrino; peristomio conico, elevato-prominente, fimbriata brunneo, circulo pallescente cincto; capillitio laxo, pallide brunneo,

floccis flexuosis, simplicibus, 3.5-4.5 μ latis; sporis concoloribus, ellipsoideis, 6-8.5 μ \times 3-4.5 μ levibus.

HAB.—Ad terram Lourenço Marques, Portuguese East Africa, legit H. A. Junod (Plates XIX and XX).

D. Junodii differs from *D. Wrightii*, as far as can be ascertained from the description of the latter, chiefly in the shape of the spores. In *D. Wrightii* the spores are described as globose, whereas in *D. Junodii* they are distinctly ellipsoid (Plate XXI, b). Lloyd (2), who records *D. Wrightii* from Bahamas and the West Indies, states that the endoperidium is "smooth"; Berkeley (1) describes it as "delicately filamentous." In our specimens it is slightly rough and floccose. Lloyd also states that the endoperidium "opens by small apertures at the top," and, in a footnote, adds: "It is not a definite, protruding mouth, as shown in figure in Engler and Prantl." Berkeley in defining the genus merely mentions "ore parvo ciliato," and in giving the specific characters, says, "opening somewhat after the fashion of *Geaster fimbriatus*." Our specimens from Portuguese East Africa exhibit a distinct fimbriated peristome (Plates XIX and XX).

The capillitium consists of fairly uniform, smooth, unbranched, wavy threads. Lloyd describes the capillitium in his specimens as made up of "various diameters from 3-30 mic., branched and interwoven. The thin shreds are almost hyaline, smooth, and not widely different from the hyaline capillitium of other gasteromycetes. The thick shreds are light yellow coloured, and, under a high power, marked with a dense reticulation."

Berkeley merely states, "Capillitium laxum." Both Berkeley and Lloyd describe the spores as globose, while the latter adds that they are "smooth or minutely punctate, many short-apiculate." The spores of *D. Junodii* are distinctly ellipsoid and smooth (Plate XXI, b).

The only other genus which in any way resembles *Diplocystis* is *Broomeia*, which was first described from South Africa in 1844 by Berkeley.

Three species have been described—viz.: *B. congregata*, Berk., and *B. ellipsospora*, V. Höhn, from South Africa; and *B. guadalupensis*, Lev., from Guadeloupe.

The correct identity of the latter species has been questioned by Lloyd, who points out that *D. Wrightii* has been found in Guadeloupe.

B. ellipsospora, V. Höhn, we have not seen, but from the description it might well have been placed in the genus *Diplocystis*, and may even be identical with our plant from Portuguese East Africa.

Both genera agree in that a number of individuals arise from a common stroma. In *Diplocystis* the stroma is rather thin and saucer-shaped (Plate XXI, a); in *Broomeia* it is usually thick and somewhat columnar (Plate XXI, c). In mature specimens of *D. Junodii* a part of the exoperidium remains as a definite coriaceous layer, the edge of the saucer enveloping each cluster of individuals; in *Broomeia* the exoperidium, when

ripe, flakes off and disappears entirely, leaving no rim to surround the individuals. In *D. Junodii* the saucer-shaped stroma is not smooth in the interior, but is thrown up into well-marked ridges, forming a separate compartment for each individual (Plate XX, Fig. 2).

In *B. congregata* this separation of the stroma into compartments is not so strongly marked and is often absent. *B. congregata* occurs frequently around Pretoria, and our examination of material at different stages of development has enabled us to confirm the accuracy of Murray's (3) description of the exoperidium of this fungus. The remains of the exoperidium on the specimens of *D. Junodii* which we have examined suggests that a similar common exoperidium covers all the individuals in this plant also. Lloyd, however, it should be noted, states that "*Diplocystis* has an individual exoperidium for each endoperidium." In conclusion, it should be mentioned that it is frequently stated that *B. congregata* grows on rotten wood. We have collected this fungus on a number of occasions in the neighbourhood of Pretoria, and have always found it growing on the ground in close proximity to the main stem of living trees of *Acacia Karroo*, Hayne (Plate XXII, Figs. 1 and 2), and have never yet found it associated with rotten wood. In several cases it has been noticed that trees thus associated with *Broomeia* were gumming freely from the main stem. An instance of this gumming is well shown on Plate XXII, Fig. 1, and at the base of the tree is an undeveloped plant of *B. congregata*. Fig. 2, Plate XXII, is a photograph of the same plant nine days later, after the exoperidium has begun to peel off.

The fructification on Plate XXIII, Fig. 1, is an exceptionally large specimen and measures 17 cm. \times 15 cm. It contained just over 900 individuals and closely embraced the stem of an *Acacia* at the ground level. The exoperidium is still present on the edges of the left-hand portion. These fungi usually appear after rain in the months of December and January, and their presence can be detected at once before they are actually found by the peculiar powerful odour which they emit, somewhat resembling aniseed.

BIBLIOGRAPHY.

- (1) BERKELEY, M. J.—"On a Collection of Fungi from Cuba," 'Journ. Linn. Soc.,' London, x, p. 344, 1869.
- (2) LLOYD, C. G.—"Mycological Writings," vol. i, p. 141, 1903; and p. 193, 1904.
- (3) MURRAY, G.—"On the Outer Peridium of *Broomeia*," 'Journ. Linn. Soc.,' London, xx, p. 311, 1883.

EXPLANATION OF PLATES XIX-XXIII.

PLATE XIX.

Specimens of *D. Junodii*, Pole Evans and Bottomley; natural size.

PLATE XX.

Specimens of *D. Junodii*, Pole Evans and Bottomley. The specimen in the lower right-hand corner shows the compartments into which the stroma is divided; all natural size.

PLATE XXI.

- a. Diagram illustrating structure of stroma of *Diplocystis*. b. Spores of *D. Junodii*, Pole Evans and Bottomley; greatly enlarged. c. Diagram illustrating structure of stroma of *Broomeia*. d. Spores of *B. congregata*, Berk.; greatly enlarged. e. Photograph of a section through *B. congregata*, Berk., showing the individual "puff-balls" embedded in the thick, white, columnar stroma.

PLATE XXII.

Fig. 1. *B. congregata*, Berk., at the base of *A. Karroo*, Hayne, showing the white exoperidium. Fig. 2. The same plant photographed nine days later, after the exoperidium has disappeared. Both greatly reduced.

PLATE XXIII.

Fig. 1. A large specimen of *B. congregata*, Berk., removed from the base of an *A. Karroo* tree. The gap in the upper portion is where the fungus wrapped almost round the stem of the tree. The specimen contained over 900 "puff-balls." Much reduced. Fig. 2. A specimen of *B. congregata*, Berk., photographed from above, and showing the exoperidium before it has begun to peel off. Natural size.

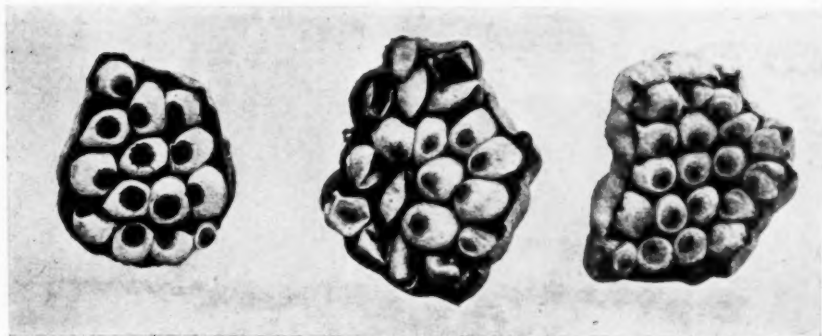


FIG. 1.

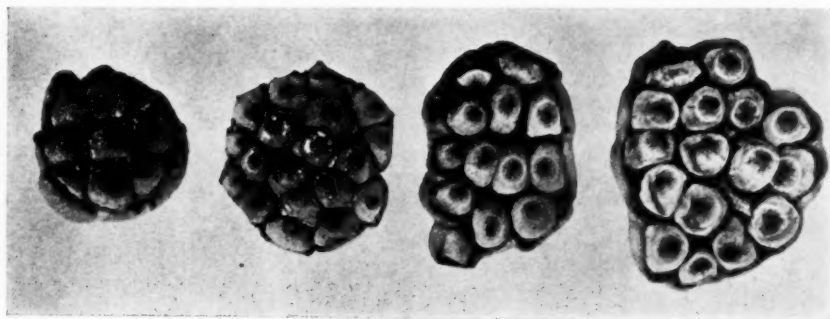


FIG. 2.

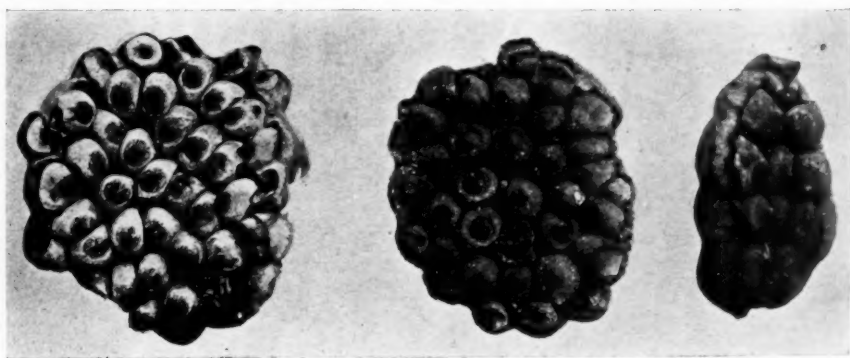


FIG. 3.

Diplocystis Jenodii, n. sp. Pole Evans and Bottomley.

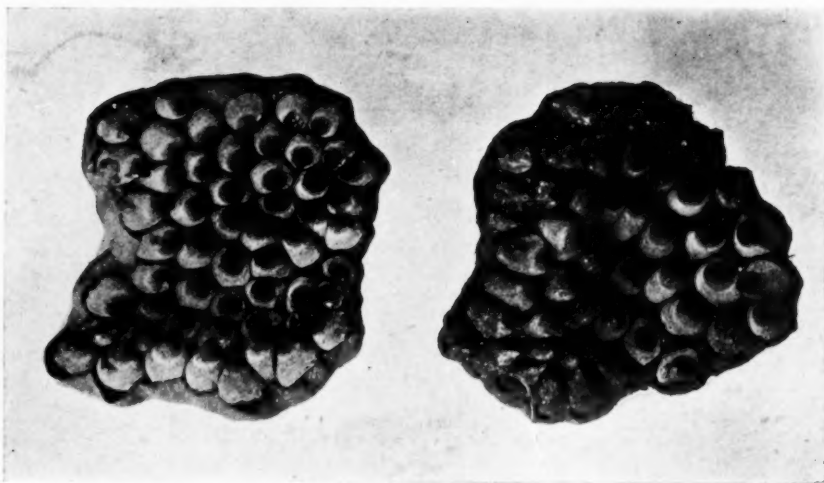


FIG. 1.

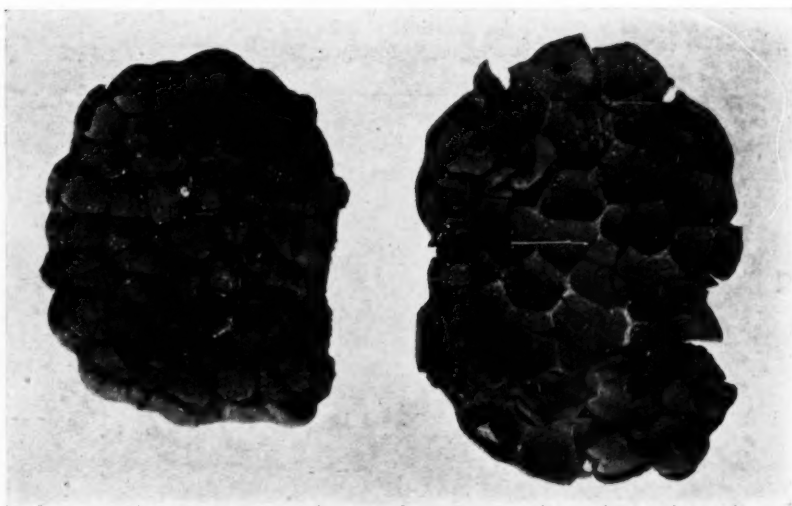


FIG. 2.

Diplocystis Janodii, n. sp.



FIG. 1.—A. Section through *Diplocystis Junodii*, n. sp. (diagrammatic). B. Spores.

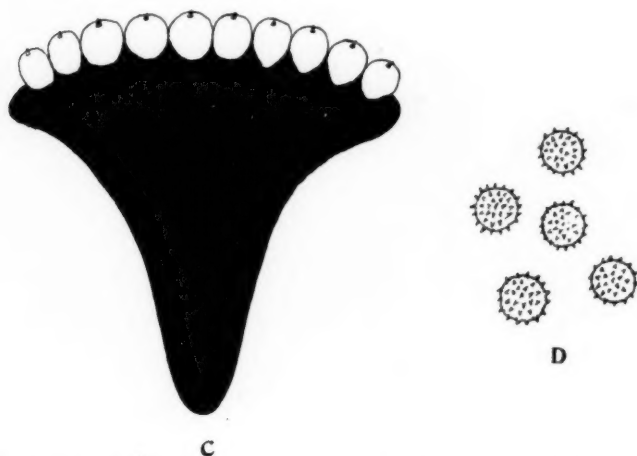


FIG. 2.—C. Section through *Broomeia congregata* (diagrammatic). D. Spores.

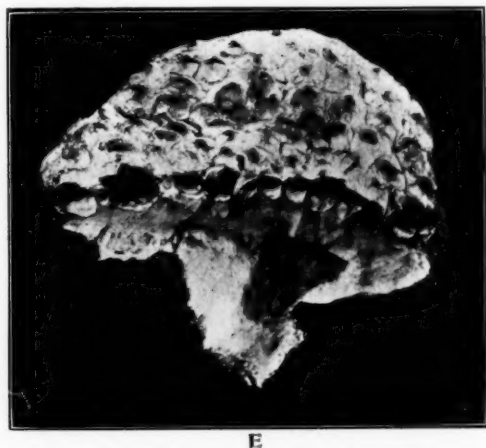


FIG. 3.—E. Section through *Broomeia congregata* (phot.).



FIG. 1.—*Broomelia conopsea* at base of *Acacia Karoo*—young stage.



FIG. 2.—Same—nine days later. (This plant contained sixty individuals.)

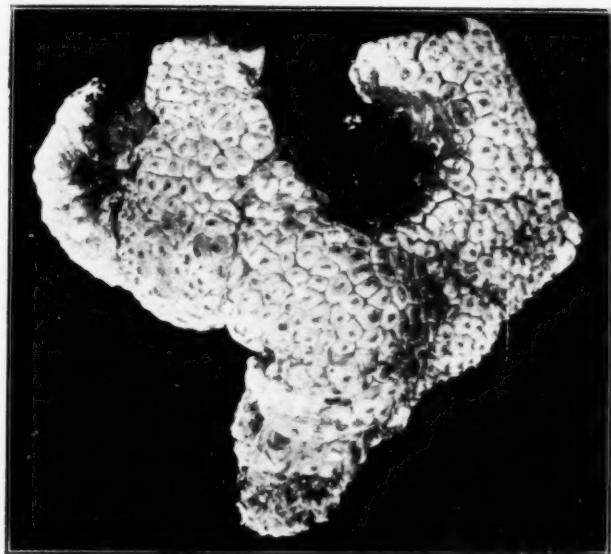


FIG. 1.

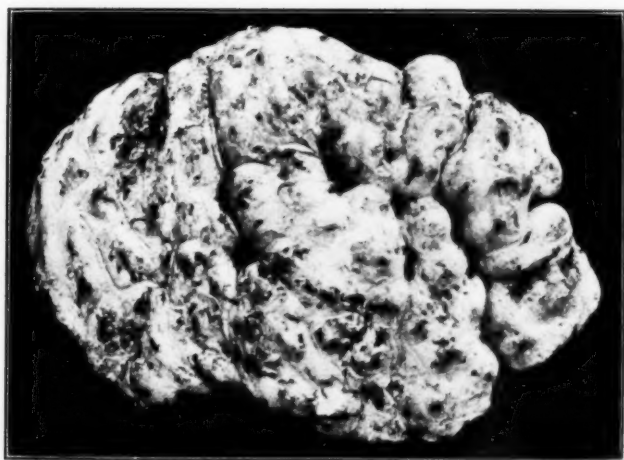


FIG. 2.

Brunmeia congregata.

SOUTH AFRICAN PERISPORIACEÆ.

II. REVISIONAL NOTES.

BY ETHEL M. DOIDGE, M.A., D.Sc., F.L.S.

(With three Text-figures.)

In working through a number of fresh collections of South African Perisporiaceæ, it has become evident that certain species described in my previous paper* on this group need revision. On p. 726 of the above-mentioned publication *Meliola torta* is described on the leaves of *Trichocladus crinitus*, and it is stated that there is another *Meliola* associated with this on the same leaves, with 4-septate spores and no setæ, but this was badly parasitised and could not be determined. In examining fresh collections of fungi on the same host, it has become evident that some confusion has arisen owing to the fact that there are at least four fungi (possibly more) on the same leaves.

One of these is an *Asterina*; of the others, in the first collection of *Trichocladus* the perithecia of *Meliola torta* were immature, and the spores of a fungus parasitic on the *Meliola*, and having similar perithecia, were described as those of *Meliola torta*. In later collections mature perithecia of *Meliola torta* were found with large 4-septate spores. It therefore becomes necessary to redescribe *Meliola torta*, and to describe the fungus parasitic on the two *Meliolas*. Mature non-parasitised material of the second species of *Meliola* was also found, and, as it appears to be a new species, I have named it *Meliola scabra*, on account of the frequently scabrous character of the capitate hyphopodia.

Meliola torta Doidge, char. emend.

Amphigena, maculas atras, tenues, 5-10 mm. diam. efficiens; hyphis tenuibus, 6-7 μ crassis, tortuosis, anastomosantibus, cellulis 25-36 μ long.; ramis irregularibus, plerumque unilateralibus; hyphopodiis capitatis stipitatis, tenuibus, stipitis nonnunquam septatis, cellula superiore 14-25 \times 14-20 μ , diverse lobata, torta, apice obtusa v. convexa; hyphopodiis mucronatis ampullaceis, 20-36 μ long.; setis mycelicis non numerosis, sparsis, rectis, simplicibus, basi 10-11 μ cr., apice acutis; peritheciis paucis, sparsis, atris,

* 'Trans. Roy. Soc. of S.A.,' vol. v, pt. 6, 1917.

globulosis, rugulosis, 250–400 μ diam. ; ascis 2-sporis ; sporidiis ellipsoideis, compressis, 4-septatis, ad septa constrictis, cellula media majore, 54–67 \times 20–22 $\mu \times$ 17 μ .

Hab. in foliis *Trichocladi criniti*, Izelini, Kingwilliamstown Dist., 8/6/15 (9064), Woodville Forest, George, 11/11/17 (10939).

Meliola scabra Doidge, n. sp.

Amphigena, maculas atras tenues 5–10 μ diam. efficiens ; hyphis, 6–7 μ

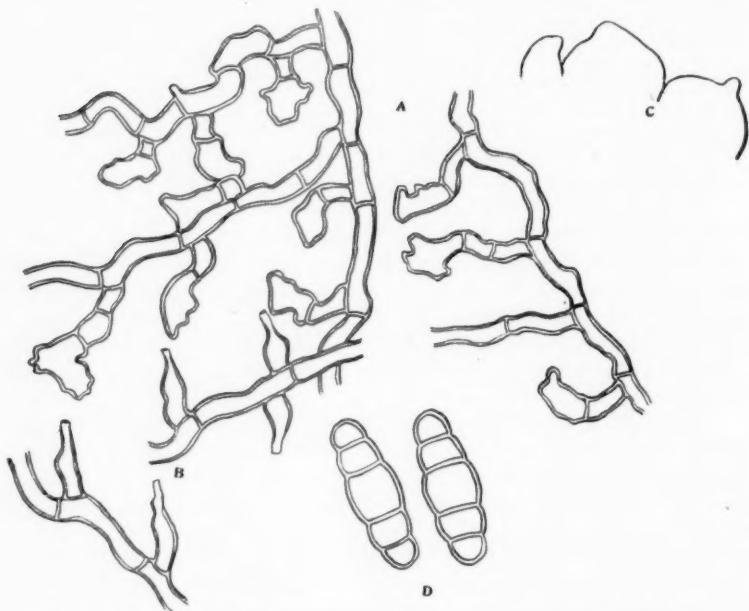


FIG. 1.—*Meliola torta*. (a) Mycelium with capitate hyphopodia ; (b) mucronate hyphopodia ; (c) superficial cells of perithecium ; (d) spores. [All figures drawn with the aid of camera lucida, with Zeiss objective D and No. 5 ocular.]

crassis, flexuosis : ramis plerumque oppositis ; cellulis 24–30 μ long. ; hyphopodiis capitatis alternis, breviter stipitatis, 23–30 μ long. cellula basali 6–7 μ long., cellula superiore ovata v. sub-uncinata, plerumque irregulariter sublobata, scabra ; hyphopodiis mucronatis inter hyphopodia capitata interspersis, oppositis v. unilateralibus, ampullaceis, apice interdum curvatis ; setis mycelicis nullis ; peritheciis atris, carbonaceis, rugulosis, 200–250 μ diam. ; ascis 2-sporis ; sporidiis 4-septatis, utrinque rotundatis, vix ad septa constrictis, 36–40 \times 14–17 μ .

Hab. in foliis *Trichocladi criniti*, Izelini, Kingwilliamstown Dist., 8/6/15, leg. Emmett (9064), Woodville Forest, George, 11/11/17, leg. E. M. Doidge (10939).

In foliis *Trichocladi elliptici*, 15/3/15, Tabankulu, Transkei, leg. G. Fraser (8891).

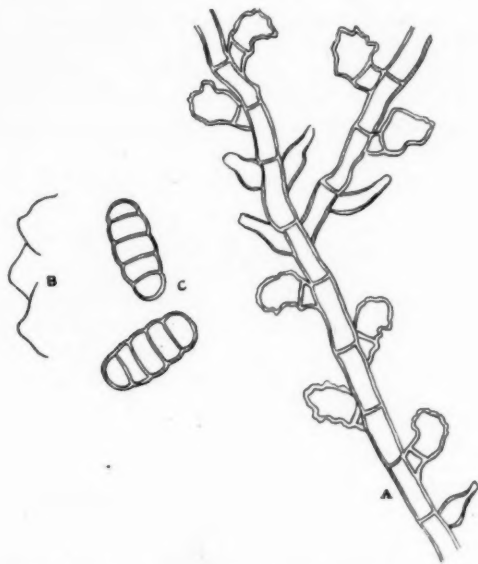


FIG. 2.—*Meliola scabra*. (a) Mycelium with both capitate and mucronate hyphopodia; (b) superficial cells of perithecium; (c) spores.

Perisporina meliolicola Doidge n. sp.

Hyphis ramosis, septatis, tenuibus, 3–3.5 μ cr., peritheciis atris, subglobois, glabris, membranaceis, 160–180 μ diam.; ascis numerosis, paraphysatis, 8-sporis, ellipsoideis-oblongis, breviter pedicellatis, 60–66 \times 18–20 μ ; paraphysibus simplicibus, filiformibus, hyalinis, ascos leniter superantibus; sporidiis distichis v. subtristichis, primum hyalinis, cellula superiore brevior, latiore, deinde, 3-septatis, clavatis, ad septa, vix constrictis, cellulis extremis valde minoribus, 25–30 \times 7–11 μ .

Hab. in mycelio *Meliolae glabrae* et *Meliolae tortae* in foliis *Trichocladi criniti*, Izelini Forest, Kingwilliamstown, 14/6/15 (9064).

Another point to which I wish to draw attention is the nomenclature of the *Meliola* commonly found on *Rubus* and also recorded on *Pygeum africanum*. The early collections of this fungus were submitted to Sydow

for determination, and he, following Gaillard,* named the fungus *Meliola manca*. In a recent publication † F. L. Stevens writes as follows:

"In 'Le Genre *Meliola*,' Gaillard writes under *Meliola manca* the three species *M. manca*, *M. sanguinea*, and *M. puiggarii*, giving a new description of *M. manca*. This new description mentions larvaform perithecial appendages. The original description of *M. manca* by Ellis and Martin mentions no such appendages, but does specifically state that there are no perithecial appendages. My own material of the two collections is ample. I have studied it carefully, also a specimen of the Heller collection, and a specimen collected by Martin and distributed by Ellis as N.A.F. No 1292, all of these upon the same host (*Myrica cerifera* L.). These specimens all agree perfectly, and agree with the original description. They do not have larvaform perithecial appendages, and therefore do not conform with the description as given by Gaillard.

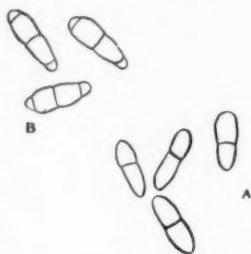


FIG. 3.—*Perisporina meliolicola*. (a) Immature and (b) mature spores.

"The specimens of *Meliola* which have been found on *Rubus* agree well with the description of *Meliola puiggarii*. They have abundant larvaform appendages, and cannot be placed under *M. manca*. Moreover, both the general characters of the mycelium and the capitate hyphopodia separate the forms on *Rubus* from those on *Myrica*, the hyphopodia on *Rubus* being much larger and more irregular in shape. I am forced, therefore, to regard the description given by Gaillard for *M. manca* as erroneous, and that of Ellis and Martin as correct, and to recognise *M. puiggarii* on *Rubus* as an entirely different species."

In view of the evidence thus clearly set forth, and after examining specimens of both species kindly sent me by Mr. Stevens, I am driven to conclude that the South African specimens on *Rosaceae* which have abundant larvaform perithecial appendages should be named *M. puiggarii*. The South African specimens agree with the Porto Rico specimens of *M. puiggarii*, the only difference being that the larvaform perithecial appendages are, if any-

* 'Le Genre *Meliola*,' p. 37.

† "The genus *Meliola* in Porto Rico," 'Illinois Biol. Monographs,' vol. ii, No. 4.

thing, less numerous and rather shorter on the South African specimens. I have also to record this fungus on a number of new hosts in addition to those already mentioned, as follows:

Meliola puiggarii Speg.

On leaves of *Rubus rigidus*, Winter's Kloof, Natal, 17/6/11, E. M. Doidge (1574); Woodbush, Zoutpansberg Dist., 5/8/11, E. M. Doidge (1771); Cramond, Natal, 3/6/12, I. B. Pole Evans (2405); Knysna, C.P., 3/6/12, P. J. Pienaar (2425).

On leaves of *Rubus pinnatus*, Buccleuch, Natal, 20/4/16, J. M. Sim (10150).

On leaves of *Pygeum africanum*, Woodbush, Zoutpansberg, 3/8/11, E. M. Doidge (1761).

On leaves of *Cliffortia strobilifera*, van Staden's Pass, 13/11/17, E. M. Doidge (10859).

On leaves of *Cliffortia ferruginea*, van Staden's Pass, 13/11/17, E. M. Doidge (10861).

On leaves of *Leucosidea sericea*, Nottingham Road, Natal, 8/4/11, P. v. d. Bijl (Natal Herbarium, No. 596).

I wish also to call attention to an error in the Explanation of Plates which was overlooked in correcting the proofs. Plate LXIII, fig. 24, is named *Meliola Peglerae*; this figure illustrates *M. inermis*; and the fig. 25, Plate LXI, which is called *M. inermis*, should be *M. Peglerae*.

NOTE ON THE ADJUGATE OF BEZOUT'S ELIMINANT
OF TWO BINARY QUANTICS.

BY SIR THOMAS MUIR, LL.D.

(1) It is manifest that the two equations

$$\begin{aligned} a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 &= 0 \\ b_0x^4 + b_1x^3 + b_2x^2 + b_3x + b_4 &= 0 \end{aligned}$$

give immediate rise to the four

$$\begin{aligned} \begin{vmatrix} a_0 & a_1x^3 + a_2x^2 + a_3x + a_4 \\ b_0 & b_1x^3 + b_2x^2 + b_3x + b_4 \end{vmatrix} &= 0, \\ \begin{vmatrix} a_0x + a_1 & a_2x^2 + a_3x + a_4 \\ b_0x + b_1 & b_2x^2 + b_3x + b_4 \end{vmatrix} &= 0, \\ \begin{vmatrix} a_0x^2 + a_1x + a_2 & a_3x + a_4 \\ b_0x^2 + b_1x + b_2 & b_3x + b_4 \end{vmatrix} &= 0, \\ \begin{vmatrix} a_0x^3 + a_1x^2 + a_2x + a_3 & a_4 \\ b_0x^3 + b_1x^2 + b_2x + b_3 & b_4 \end{vmatrix} &= 0; \end{aligned}$$

and that these when expressed explicitly as cubics in x furnish us at once with

$$\begin{vmatrix} |a_0b_1| & |a_0b_2| & |a_0b_3| & |a_0b_4| \\ |a_0b_2| & |a_0b_3| + |a_1b_2| & |a_0b_4| + |a_1b_3| & |a_1b_4| \\ |a_0b_3| & |a_0b_4| + |a_1b_2| & |a_1b_3| + |a_2b_2| & |a_2b_3| \\ |a_0b_4| & |a_1b_4| & |a_2b_3| & |a_3b_4| \end{vmatrix} = 0,$$

the left-hand member of which is known as Bezout's eliminant.

(2) To the adjugate of this eliminant considerable study has been given since Jacobi first drew attention to it ('Crelle's Journ.," xv, pp. 101-124). Unfortunately there has been no simple mode of expressing its elements, which rapidly increase in complexity with the degree of the determinant. Taking advantage of the fact much later established that the primary minors of Bezout's eliminant have equivalents among the secondary minors of Sylvester's eliminant, I have succeeded in obtaining for the adjugate an expression whose law of formation is perfectly simple. Before stating it, it is necessary to recall the fact that the most convenient form of Sylvester's eliminant is that in which the rows of b 's follow the opposite order of the rows of a 's, being in the case which we are considering

$$\begin{vmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & . & . & . \\ . & a_0 & a_1 & a_2 & a_3 & a_4 & . & . \\ . & . & a_0 & a_1 & a_2 & a_3 & a_4 & . \\ . & . & . & a_0 & a_1 & a_2 & a_3 & a_4 \\ . & . & . & b_0 & b_1 & b_2 & b_3 & b_4 \\ . & . & b_0 & b_1 & b_2 & b_3 & b_4 & . \\ . & b_0 & b_1 & b_2 & b_3 & b_4 & . & . \\ b_0 & b_1 & b_2 & b_3 & b_4 & . & . & . \end{vmatrix};$$

and that the most convenient way of specifying any minor of this eliminant is by giving the numbers of the rows and columns from which the minor is taken; for example,

$$\begin{vmatrix} 237 \\ 357 \end{vmatrix} \text{ standing for } \begin{vmatrix} a_1 & a_3 & . \\ a_0 & a_2 & a_4 \\ b_1 & b_3 & . \end{vmatrix}.$$

(3) *The adjugate of Bezout's eliminant of*

$$a_0x^4 + a_1x^3 + \dots + a_4 = 0, \quad b_0x^4 + b_1x^3 + \dots + b_4 = 0$$

is

$$\begin{vmatrix} \mu_1 & \mu_2 & \mu_3 & \mu_4 \\ \mu_2 & \mu_3 & \mu_4 & \mu_5 \\ \mu_3 & \mu_4 & \mu_5 & \mu_6 \\ \mu_4 & \mu_5 & \mu_6 & \mu_7 \end{vmatrix}$$

where

$$\mu_1 = \begin{vmatrix} 234567 \\ 345678 \end{vmatrix}, \mu_2 = \begin{vmatrix} 234567 \\ 245678 \end{vmatrix}, \dots$$

the row-numbers of μ_r being 234567 and the column-numbers being obtained by deleting the digit $r + 1$ from 2345678.

To prove the identity of any one of the μ 's with the appropriate primary minor, we may follow the procedure laid down by Le Paige in 1880 ('Comptes Rendus' . . . Paris, xc, pp. 1210-1212), namely, multiply the μ in question by such another form of itself as will give the square of the desired equivalent; for example,

$$\begin{aligned} -\mu_1^2 &= -\begin{vmatrix} 234567 \\ 345678 \end{vmatrix}^2 = -\begin{vmatrix} 234567 \\ 345678 \end{vmatrix} \cdot \begin{vmatrix} 567234 \\ 678345 \end{vmatrix}, \\ &= \begin{vmatrix} 234\bar{5}67 \\ 345678 \end{vmatrix} \cdot \begin{vmatrix} 567234 \\ 678345 \end{vmatrix}, \end{aligned}$$

where the minus above a row-number indicates change of sign in the row; and columnwise multiplication now gives

$$-\begin{vmatrix} |a_0b_3| + |a_1b_2| & |a_0b_4| + |a_1b_3| & |a_1b_4| \\ |a_0b_4| + |a_1b_3| & |a_1b_4| + |a_2b_3| & |a_2b_4| \\ |a_1b_4| & |a_2b_4| & |a_3b_4| \end{vmatrix},$$

as desired.

It will be found, however, less stale and more effective to multiply the determinant

$$\begin{vmatrix} 1 & . & . & . & . & -b_0 \\ . & 1 & . & . & -b_0 & -b_1 \\ . & . & 1 & -b_0 & -b_1 & -b_2 \\ . & . & . & a_0 & a_1 & a_2 \\ . & . & . & . & a_0 & a_1 \\ . & . & . & . & . & a_0 \end{vmatrix}$$

columnwise by the 6-by-7 array $\begin{vmatrix} 234567 \\ 2345678 \end{vmatrix}$, for then we obtain

$$a_0^3 \cdot \begin{vmatrix} 234567 \\ 2345678 \end{vmatrix} = \begin{vmatrix} a_0 & a_1 & a_2 & a_3 & a_4 & . & . \\ . & a_0 & a_1 & a_2 & a_3 & a_4 & . \\ . & . & a_0 & a_1 & a_2 & a_3 & a_4 \\ . & . & . & |a_0 b_1| & |a_0 b_2| & |a_0 b_3| & |a_0 b_4| \\ . & . & . & |a_0 b_2| & |a_0 b_3| + |a_1 b_2| & |a_0 b_4| + |a_1 b_3| & |a_1 b_4| \\ . & . & . & |a_0 b_3| & |a_0 b_4| + |a_1 b_3| & |a_1 b_4| + |a_2 b_3| & |a_2 b_4| \end{vmatrix},$$

the seven results of which give equivalents for all the μ 's, and in four cases ($\mu_4, \mu_5, \mu_6, \mu_7$) the exact equivalents wanted.

(4) What has just been established for the eliminant of the fourth order holds equally for any other order, and the mode of proof is quite general. In case of any doubt about the formation of the μ 's, it need only be added that when the eliminant is of the n^{th} order, the row-numbers of μ_r are 2, 3, 4, . . . , $2n-1$, and the column-numbers are obtained from 2, 3, 4, . . . , $2n-1, 2n$.

(5) Le Paige's process, mentioned in § 3, led subsequent writers to the discovery of the fact that Bezout's eliminant is variously expressible as the result of multiplying n columns of Sylvester's eliminant by a transformation of the other n columns.* This suggests the making of a similar

* So far as I am at present aware, the first publication of this is due to H. W. Tyler ('Sitzungst. d. phys.-med. Soc. zu Erlangen,' xxiii, pp. 33-128). He makes an oversight, however, in saying that the first set of n columns must be consecutive, and therefore in saying that the number of different pairs of factors is $n+1$. The number in question is either n or $2n$, according to the point of view. Thus, in the case where n is 3 there are the following equivalents of Bezout's eliminant:

$$\begin{vmatrix} 123456 \\ 123 \end{vmatrix} \cdot \begin{vmatrix} 654321 \\ 456 \end{vmatrix}, \quad \begin{vmatrix} 123456 \\ 456 \end{vmatrix} \cdot \begin{vmatrix} 654321 \\ 123 \end{vmatrix}, \\ \begin{vmatrix} 123456 \\ 234 \end{vmatrix} \cdot \begin{vmatrix} 465132 \\ 156 \end{vmatrix}, \quad \begin{vmatrix} 123456 \\ 156 \end{vmatrix} \cdot \begin{vmatrix} 465132 \\ 234 \end{vmatrix}, \\ \begin{vmatrix} 123456 \\ 345 \end{vmatrix} \cdot \begin{vmatrix} 546213 \\ 126 \end{vmatrix}, \quad \begin{vmatrix} 123456 \\ 126 \end{vmatrix} \cdot \begin{vmatrix} 546213 \\ 345 \end{vmatrix},$$

the last two in the second column being evidently as worthy of enumeration as the first in the column.

deduction in regard to the primary minors of the eliminant, our first proof regarding μ , in § 3, giving us at once

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_0 & a_1 & a_2 \\ & a_0 & a_1 \\ & -b_0 & -b_1 \\ -b_0 & -b_1 & -b_2 \\ -b_1 & -b_2 & -b_3 \end{vmatrix} \cdot \begin{vmatrix} b_2 & b_3 & b_4 \\ b_3 & b_4 & . \\ b_4 & . & . \\ a_4 & . & . \\ a_3 & a_4 & . \\ a_2 & a_3 & a_4 \end{vmatrix} = \begin{vmatrix} |a_0b_3| + |a_1b_2| & |a_0b_4| + |a_1b_3| & |a_1b_4| \\ |a_0b_4| + |a_1b_3| & |a_1b_4| + |a_2b_3| & |a_2b_4| \\ |a_1b_4| & |a_2b_3| & |a_3b_4| \end{vmatrix},$$

$$\text{i. e.} \quad \begin{vmatrix} 234567 \\ 345 \end{vmatrix} \cdot \begin{vmatrix} 567234 \\ 678 \end{vmatrix} = \mu_1;$$

and at the same time

$$\begin{vmatrix} 234567 \\ 678 \end{vmatrix} \cdot \begin{vmatrix} 567234 \\ 345 \end{vmatrix} = -\mu_1.$$

The theorem obtained as the result of this suggestion is that *the adjugate of Bezout's eliminant of*

$$a_0x^4 + a_1x^3 + \dots + a_4 = 0, \quad b_0x^4 + b_1x^3 + \dots + b_4 = 0$$

is

$$\begin{vmatrix} v_1 & v_2 & v_3 & v_4 \\ v_2 & v_3 & v_4 & v_5 \\ v_3 & v_4 & v_5 & v_6 \\ v_4 & v_5 & v_6 & v_7 \end{vmatrix},$$

where

$$v_1 = \begin{vmatrix} 234567 \\ 345 \end{vmatrix} \cdot \begin{vmatrix} 567234 \\ 678 \end{vmatrix},$$

$$v_2 = \begin{vmatrix} 234567 \\ 245 \end{vmatrix} \cdot \begin{vmatrix} 567234 \\ 678 \end{vmatrix},$$

.....

and the formation of v_r from μ_r is readily apparent.

RONDEBOSCH, S.A.,

January 11th, 1918.

LUMINOSITY AND ITS ORIGIN IN A SOUTH AFRICAN
EARTHWORM (*CHILOTA* SP.?).

By J. D. F. GILCHRIST, M.A., Ph.D., D.Sc.

(With Plate XXIV.)

Cases of luminosity or phosphorescence in earthworms have from time to time been observed. From 1670 (Grimm) up to the present day there are records of such observations. Vejdovsky (1884) found the common earthworm *Allolobophora foetida* to give out an occasional phosphorescent light, from the glandular cells of the epidermis he believed; and Owianikow (1864) observed *Enchytraeus albidus*, probably identical with *Lumbricus noctilucus* of Eversmann (1838), to show a light, which appeared, sometimes on the head region, sometimes on the tail, and, at other times, over the whole body. According to Flaugergues (1771), the light occurs chiefly at the region of the clitellum, and disappears after copulation. Moquin Tandon and Panceri (1875) also believed that the clitellum is the source of the luminosity. Giard (1887), on the other hand, found small, luminous worms, which he calls *Photodrilus*, and indicates that the source of luminosity is in certain glands, which surround the alimentary tract, and open on the back of the animal to the exterior. In addition to the authors quoted, others, such as Dugès (1837), Moniez, Stein (1831), Matzdorff (1883), Haupt (1903), etc., also mention luminosity in earthworms (see references).

It does not appear, therefore, that there can be any doubt as to the occurrence of the phenomenon, though there is much diversity of statement as to the facts and the probable cause of the luminosity. In view of this uncertainty, the most recent view on the subject is that the luminosity is brought about secondarily, and is probably due to the presence of luminous bacteria or fungi in the body of the earthworm. Mangold (1910) suggests luminous bacteria or fungi, and he remarks that it is strange that a peculiarity of such a striking nature in an animal so common as the earthworm should be so little known. The latest reference is contained in Dahlgren's account of luminous animals (1916). While he finds many accurate and exhaustive accounts of luminosity or luminous organs in other animals, including the marine worms, he finds nothing definite to record in the case of earthworms, and is inclined to believe that the luminosity is due to the accidental presence

of luminous fungi in the food of the animal. The presence of luminous bacteria or fungi seems a most probable explanation, and appears to be generally accepted, but, maybe, it is just because of this plausibility that the phenomenon has not been adequately investigated.

There does not appear to be any record of such an occurrence in S. Africa, and yet it does not seem to be uncommon, for several correspondents have assured me that luminous "worms" have been observed in up-country districts. In some of these cases it is possible that the common glow-worms or centipedes may have been mistaken for worms. More convincing evidence has been obtained, however, nearer Cape Town. Mr. MacManus has, he informs me, frequently seen earthworms on damp nights crawling up on the verandah of his house, near the foot of Signal Hill, leaving a trail of luminous substance; on a dark, wet night at Salt River, a patch of luminous material was observed, which was given out by an earthworm; a collector of material for the Zoological Department of the S. African College has frequently seen luminous worms at Maitland, and my friend, Dr. Purcell, informs me of an interesting case near Hermanus on the south coast. His attention was first attracted by observing the phosphorescence on a number of small toads in the grass after rain. Luminous fragments of earthworms were found on the spots occupied by the toads. Dr. Purcell was not sure that the toads had partly eaten the worms, but he attributed their luminosity to contact with the worms. The toads, on crawling away, left a luminous trail behind them; he also informs me of another instance, in which he observed luminous patches on a spider, found in the grounds of the South African Museum. These patches were readily brushed off, and were believed to have been caused by contact with some luminous animal, probably a worm.

That luminous earthworms are widely distributed in this country is rendered probable by an observation of Mr. E. J. O'Connor, who informs me that he has often seen them in damp weather at Limbe in the Shire highlands of Nyasaland.

The occasion for these inquiries was the finding of several examples of a species of earthworm (apparently a species of *Chilota*), which certainly did produce a very striking display of light. The first specimen found was on July 19th, 1917, after an exceptionally heavy rain. It was found at dusk on a footpath through the pine-woods on the slopes of Table Mountain. The light given off was conspicuous, and could not have been overlooked by the most casual observer. A few yards further on another specimen was observed. In the first case there were patches of a luminous substance a few inches from each other, as observed in the case mentioned by Giard. In the second case these were closer together, and within an area which could be covered by the palm of the hand. The general appearance was not unlike that shown in Fig. 1, which was obtained by placing a worm on a photographic plate in the dark. The irritation of the dry plate caused the animal

to give out the luminous fluid, and, on development, the luminous patches appeared as shown.

That the occurrence is not a usual one in this locality is evident from the fact that this particular path had been traversed almost daily, at all seasons of the year, and in all weathers, without any sign of the presence of such worms.

These earthworms were apparently not in the normal condition, having probably been driven out of their burrows by the unusual wetness of the soil. An endeavour was therefore made to find them in their natural habitat, and in sufficient quantity to afford material for more careful examination. The spot where they were found was dug up to a considerable depth, but none were discovered. At other places various species of earthworms were readily found, but were not luminous. On continuing the search, however, a species was dug up, which gave out a distinct light on handling, and ultimately several spots were found, where the worm could be got in sufficient abundance.

How the Light is Given Off.

It was not at first observed from what part of the animal the light was produced. In the first example procured it was found on the head and tail regions, as well as on other parts of the body. Subsequent and repeated experiments, however, seemed to indicate clearly that it was given off only from the mouth and anus, except in those cases when there was some injury to the body. The luminosity proceeded from a discharge, which was sometimes of a viscid, mucus-like consistency, sometimes of a more fluid nature. This discharge was given out in various ways, but only after a certain amount of irritation. When first dug up the worm usually assumed a rigid attitude, and could be handled freely without exhibiting any movement—perhaps a death-feigning device. Usually, however, after a time, it began to exhibit very lively movements, and, by a series of strong flexures of the body, to throw itself about, scattering masses of luminous substance in all directions (Fig. 1). If held firmly to prevent such movements the fluid could be seen to proceed from the mouth, and, to a less extent, from the anus. It usually flowed out somewhat suddenly and spread over the body, but, in several instances, it was ejected with considerable force to some distance. In many instances, when examined in the dark room, spots of light appeared unexpectedly in various places, usually on the clothes of the observer. The clear pear-shaped patch in Fig. 1 is apparently produced by such an ejected mass of luminous fluid.

The diversity of statement in other cases as to the region of the body on which the light appears may be due to the accidental contact of the body with the discharged mucus when the animal is captured, as was the case here. If, however, the animal is handled carefully, it may be washed in

water, and even dried subsequently on blotting-paper without exciting any discharge. A worm treated in this way was placed between two photographic plates in a dark room. The adhesion of the dry surface of the film prevented any movement of the animal, and, at the same time, induced a discharge. The details of this could be watched through the plate. A light spot first appeared at the mouth, and slowly spread backward over the surface of the anterior segments of the animal. Similarly, but some time afterwards, a light spot appeared at the anus, spreading on to the surface of a few of the posterior segments. The earthworm was then removed, and on developing these light areas appeared as a blacking of the plate (Fig. 2). In another case the spreading of the fluid was allowed to go further, and the whole of the body of the worm ultimately became enveloped in the luminous substance. Here also the light was observed to appear first at the mouth and anus, and thereafter to spread over the body. To obtain further information as to the source of the luminosity, a worm was compressed between two plates, before the luminous fluid had time to spread along the body from the mouth and anus. The result is seen in Fig. 3, in which it will be noted that the luminous fluid has been forcibly ejected at the various parts where the body has been ruptured. In interpreting this result, it must be remembered that, after the compression, there was time for the fluid to spread to some extent between the closely applied plates elsewhere than at the place where the fluid was first ejected. It was at first supposed that the luminous substance had been ejected from the alimentary canal, but, as will be shown later, its actual seat is in the body cavity.

The Luminous Fluid.

When first extruded the luminous fluid appeared of a whitish colour in daylight, and could not well be detected, except with careful observation under a lens. If shaded by the hand, however, it could be seen to be of a greenish colour, and to be luminous. When examined under the microscope it was found to consist of a great number of cells, so densely packed together as to be in contact with each other. Occasionally a greater or less amount of a viscid mucus occurred along with these, but, though the cells were more or less entangled in this mucus, it apparently was not necessarily associated with them. There was also present in all cases a certain amount of clear, watery fluid.

The greater majority of the cells were of a large size, the largest being about 20μ in diameter. These were very heavily laden with inclusions, most of which were rounded, apparently of a fluid nature, and of a greenish colour. These were also of various sizes, the largest being about the size of the nucleus of the cell. Scattered among these were small specks or granules, apparently of a more solid nature.

The most of these cells showed no movement, but a slow movement could

be detected in some, and, in a few, this was well marked. After a time all these cells became disintegrated, and, soon after the commencement of this process, the whole of the contents of the cell became scattered about in the surrounding clear fluid.

Other cells, fewer in number and smaller, were also found, circular in outline with a clear border and small inclusions in the centre, and appeared to be earlier stages in the formation of the large cells.

More numerous, and entirely devoid of inclusions, were numbers of much smaller cells, with relatively large nucleus and irregular outline. Some of these showed active movement, when observed in the fresh condition, either in a hanging drop or under a cover-glass.

In addition to these there were frequently observed gregarines of at least four different species. In several instances there were also present the typical pseudo-navicellae or spores of the gregarines, and these were often seen associated with groups of the smaller cells above referred to.

Nature and Cause of the Luminosity.

The amount of luminous fluid discharged varies very considerably, both according to the extent of the irritation and the condition of the animal, but the intensity of the luminosity appeared to be always in proportion to the quantity of the fluid. A quantity, about the size of a pin's head, when placed on a slide, maintained its luminosity for a few minutes only, but, when the discharge was abundant, as, for instance, when it covered an area the size of a sixpence or even overflowed the slide, it retained its luminosity for over an hour. When a small quantity was at once shaken up in about 20 c.c. of water, the water became faintly luminous, and retained its luminosity for over two hours. The intensity of the luminosity may be judged by the fact that moderately large print could be made out in the dark room when close to the light. When left on a fairly slow photographic plate (Ilford ordinary) for about half a minute, the blacking of the plate proceeded on development in a normal way for correct exposure.

The luminous fluid could be examined microscopically in a dark room by the aid of its own light. This examination did not, however, at first prove so instructive as might have been expected, for the light given off presented the appearance of a uniform glow without any differentiation. This was due apparently to the fact that the light was produced during the process of the breaking up of the granular cells already mentioned, and the scattering of their contents. That these contents were still capable of giving out light after discharge was evident from the fact that light was produced after all the cells had been disintegrated, and that the luminous fluid, after being shaken up in water and passed through filter paper, was still luminous.

Though the fluid was repeatedly examined under the microscope with varying magnifications without any differentiation being detected, on one

occasion, after prolonged observation, and when the fluid was almost dried up, innumerable minute specks of bright light were plainly seen. These were well defined, each standing out distinctly on the dark background as a bright star-like glow. The glow of each particle did not, however, last for more than six to ten seconds, though some remained distinct for about thirty seconds. The light appeared first as a faint speck, and gradually reached its maximum as a steady glow, when it faded off rather suddenly. This appearance was not readily observed in this way, and often the light died down uniformly. Another method, however, by means of which the luminous particles could be readily distinguished was subsequently found, when it was accidentally discovered that the fluid so dried up could be again made luminescent by the application of water. If a quantity of water be put on the slide, it was noticed that, at the point where it came in contact with the dried substance, a distinct glow appeared, and this, when examined in the dark under the microscope, was seen to consist of numerous, discrete, luminous particles. As the water encroached on the dried substance, an area of bright luminous specks, like a starry sky, passed slowly over the field of vision, at the point of contact of the water and dried substance. Subsequently it was found that the luminosity could also be renewed simply by breathing gently on the dried slide, but in this case it was of very short duration. The experiment was then tried of drying the slide rapidly, by subjecting it to a temperature of about 60° C., when the luminosity again reappeared on the application of moisture.

It was, of course, impossible to locate the exact position of these luminous particles with reference to the cells in the fluid, which were invisible in the darkness, but by turning on an artificial light after their position in the field of vision had been noted, it was observed that they were situated in the cells, but that all of the cells were not luminous, or at least did not give out light sufficiently strong to be seen through the lenses of a microscope. It was only on one occasion, however, that three such isolated cells were observed to have these luminous particles, and then they were apparently in the process of breaking down.

The luminous particles were found amongst the discharged granules of the cells in abundance.

The size of the luminous particles could not be accurately ascertained, but they were of about the same size as the small particles already mentioned as occurring in the cells, and there is good reason for believing that they were identical with them. Such solid particles are known to occur in the cells of the body cavity, and are believed to be the result of the transformation of the fluid spheres, which are purely excretory matter. Thus Kükenthal and Joseph describe in *Tubifex* and *Lumbricus* the large brown masses in the chloragogen cells as being transformed into small solid and dark particles. In the present case, however, these particles appear to be of

fatty nature, as they stain with osmic acid and Sudan III. But, though there seems now to be a general agreement that the chloragogen cells contain only excretory matter, the presence of fat has been detected in those of some *Limicola*, especially the family of the *Enchytraeidae* (Rosa, Freudweiler). Willem has found fat globules in the chlorogogen cells of *Arenicola*, and Ashworth in those of the Polychaete *Halla*.

If these particles be of the nature of fat, the luminescence is sufficiently explained, as being merely a process of oxidation, as it is known that some substances of this nature react in this way.* As further evidence that the luminosity is caused by oxidation, I may mention that a quantity of the fluid, left on a slide under a cover-glass for a few days, was observed to give out light on removal of the cover-glass and exposure to the air. In one case the fresh material had been observed under a cover-glass ringed with vaseline to exclude air; after examination the slide was put aside, and, about a month afterwards, on removing the cover-glass to clean the slide, the fluid was observed to be distinctly luminous, both on slide and cover-glass. Another slide of the same kind was kept about two months, but gave out no light when the cover-glass was removed. It would thus appear that a slow process of oxidation takes place, during which no visible light is given off. Further evidence of this is that if the fluid be dried on the slide, and left for a few hours exposed to the air, it does not become luminous on the application of water or moisture. It may be noted also that if the luminous fluid be rubbed on a cloth or other substance the luminosity is increased, but dies down quickly.

The test for luciferin and luciferase was applied, but these substances are apparently not present.

Origin of the Luminous Cells.

The cells, laden with inclusions of various sizes, might be the cells given off from some tissue of the earthworm itself, or cells of some parasitic protozoon undergoing rapid multiplication. They resemble most the free cells found in the body cavity of the earthworm, either chloragogen cells, or amoebocytes, laden with excretory matter; but such cells are not known to find access to the digestive tract, though it may be noted that at first they were believed to form digestive ferments (D'Udekem)—a supposition which was abandoned when it was recognised that it was "impossible that the contents of the chloragogen cells could be poured into the lumen of the alimentary tract" (Burian).

This being so, it was apparently impossible that the cells discharged from the mouth could be cells from the body cavity. It was therefore conjectured that they were some stage in the life-history of a protozoon para-

* E. Newton Harvey, "Studies in Bioluminescence," 'American Journal of Physiology,' vol. xli, No. 4, 1916.

site, and the presence in several cases of numerous "pseudonavicellae" seemed to indicate that this parasite was a gregarine.

It was found, however, on making sections, that the cells undoubtedly proceeded from the body cavity by a comparatively large aperture on the floor of the buccal cavity; posteriorly also, within or at the edge of the anus, a small aperture was found on the dorsal aspect leading into the body cavity. There could be little doubt on this point, for sections were made of a worm which had just discharged the luminous fluid from the mouth, and in the sections the remains of this fluid in the mouth cavity could be clearly followed through the aperture to the body cavity. The structure and nature of these openings will doubtless prove of interest, but we are not concerned with them here further than that they throw light on the origin of the luminous cells.

It was expected that the coelomic fluid and its contained cells would be ejected by the dorsal pores, as was found to be the case in a species of *Lumbricus* obtained from the same locality, when placed in a preservative. This, however, was not the case; indeed, dorsal pores seem to very inconspicuous or absent in this species of *Chilota*, as none were seen.

The luminous cells therefore originate from the body cavity, and are apparently either free chloragogen cells or amoebocytes, or, maybe, a mixture of these.

Biological Significance of the Luminosity.

So striking a feature as the production of light by an animal naturally suggests the likelihood of a specific purpose for which the light is produced. There is usually, however, a considerable difficulty in assigning to it any definite function with certainty, even in those animals with distinct and somewhat complicated organs for the production of light. Thus, for instance, while it has been maintained with a good deal of reason that the light produced by the fire-fly, certain fishes and crustacea are secondary sexual characters, this has been disputed. Certain luminous organs are apparently for the attraction of the prey of the animal, for scaring off or misleading enemies; but while the complexity and apparent purposefulness of the organs undoubtedly point to a definite function, the proof of this, founded on facts, is markedly wanting. If this be the case in the higher animals it is much more so in the lower. Luminous protozoa such as *Noctiluca*, so well known, luminous bacteria, equally well known, have no obvious "use" for their luminosity. There is no difficulty in suggesting a plausible use for the luminosity in the earthworm. Thus, when this luminous worm was first found crawling about on the surface of the ground in the semi-darkness, the patches of luminous mucus, while so readily attracting attention, apparently served the purpose of drawing off attack from the worm itself, as it actually did in the observed cases—a common enough expedient in some animals, which sacrifice a conspicuous organ by throwing it off from

the body. The utility of luminosity is apparently still more obvious in the underground life of the animal. Thus it is guarded in its burrow from attacks both in front and in the rear, and any predaceous insect, centipede, or other enemy, on attacking the worm at either end, would be deluged in a blinding mass of luminous fluid, which, in the darkness of underground life, would be sufficiently alarming to cause a hasty retreat.

The behaviour of the worm also seems to indicate a defensive organ of this nature, for it most readily and in every case gives off the luminous fluid if the head or tail is slightly touched by a sharp pin or needle. The fluid was, as stated, sometimes ejected to a considerable distance, and recalled the similar, but much more definite, ejection of a mucous substance by *Peripatus*.

The facts narrated suggest several other promising subjects of investigation for workers in S. Africa. Other species of earthworm in S. Africa would probably be worth investigating from this point of view. For instance, in digging for worms in a garden a species was found, but only one example, which gave out an abundant discharge from the mouth, consisting mainly of cells such as described but showing no luminosity. Again the chemical nature of the luminosity is a promising line of research, as the material is so readily got and manipulated; the exact nature and origin of the cells, which occur in such numbers, is a subject worth investigating; and, lastly, the structure, origin, and possible homology of the communications between the coelom and digestive tract.

I have to express my indebtedness for assistance in literature to my colleagues, Prof. Jolly and Mr. Van der Lingen; to Dr. Robertson, Government Bacteriologist, for examination of the luminous fluid for the presence of bacteria; and to Dr. Battaerd for confirmation of the fatty nature of the small granular inclusions of the cells.

Summary.

(1) A few examples of a species of luminous earthworm were found on the surface after heavy rains, and subsequently in quantity by digging in the ground.

(2) The luminosity proceeds from a fluid discharged from the mouth, and, to a less extent, from the anus.

(3) The luminous fluid consists mostly of single cells heavily laden with inclusions of different kinds.

(4) The luminosity is given out by the inclusions of small size, and these seem to be of a substance allied to fat, by the oxidation of which the light is produced.

(5) The cells arise in the body cavity, and are discharged into the anterior and posterior ends of the alimentary tract by definite communications between coelom and alimentary tract.

REFERENCES AND BIBLIOGRAPHY.

1670. GRIMM.—Quoted by Friend, 1893.
1771. FLAUGERGUES.—“Lettre sur le phosphorisme de vers de terre,” ‘Journ. de Phys.,’ t. 16 (1771), p. 311.
1792. BEUGIÈRE.—“Sur la qualité phosphorique du ver de terre dans certain circonstances,” ‘Journ. d’Hist. nat.,’ t. 2, p. 267.
1831. STEIN, F. V.—“Ueber einen neuen das leuchten der ostsee bedingenden lebenden Körper,” ‘Poggendorff’s Ann. d. Phys. u. Chem.,’ Bd. 33, p. 410 (1831).
1837. DUGÈS, ANT.—“*Lumbricus phosphoreus*,” ‘Ann. Sc. Nat.,’ ser. 2, vol. 8, pp. 17, 24.
1838. EVERSMAAN.—“*Lumbricus noctilucus*,” ‘Utchen. Zapiski. Kazan. Univ.,’ 1838, pp. 156–157 (Russian).
1843. ALLMAN showed at Cork meeting of British Association luminous earthworm from bogs of South of Ireland, ‘Report of British Association,’ 1843, p. 76.
1853. COX showed luminous earthworm at a meeting of Lit. and Phil. Soc., Liverpool. ‘Proc.,’ No. viii, p. 57.
1864. OWSIANNIKOW, PH.—“Ueber das Leuchten der Larven von *Lampyrus noctiluca*,” ‘Bull. acad. imp. sc. St. Petersburg,’ t. 7, p. 55.
1872. SECCHI.—‘Nouv. observ. Ann. Sc. Nat.,’ ser. 5, vol. xvi, 1872.
1873. COHN.—“Leuchtende Regenwürmer,” ‘Zeitschrift für Wiss. Zool.,’ vol. xxiii, pp. 459–461.
1875. PANCERI, P.—“La luce e gli organi luminosi annalidi,” ‘Atti. della R. Accad. d. Sc. fis. e nat.,’ vol. 7, No. 1.
1884. VEJDovsky.—‘System u. Morphologie der Oligochaeten. Prag.’
1887. GIARD.—“*Photodrilus phosphoreus*,” ‘C. R. ac. sc.,’ vol. 105, p. 872.
- HARKEE.—“On a Luminous Oligochaete (*Enchytraeus*),” ‘Rep. Brit. Ass.,’ 1887, p. 767.
- ATKINSON, G. F.—“A Remarkable Case of Phosphorescence in an Earthworm,” ‘American Naturalist,’ vol. xxi, pp. 773–774.
1891. BARROIS, TH.—“Sur le présence de *Lumbricus (Photodrilus) phosphoreus*, Dugès, à Groffliers (Pas de Calais),” ‘Rev. biol. Nord France,’ t. 3, No. 3, pp. 117–119.
1893. MATZDORFF.—“In Berlin aufgefunden leuchtende Regenwürmer,” ‘Sitzb. d. Ges. Naturf. Freunde,’ 1893, pp. 19–23 (gives synopsis of previous records).
- FRIEND, H.—“Luminous Earthworms,” ‘Nature,’ vol. 47, 1893, p. 462.
1897. ——— “Earthworm Studies: Phosphorescence and Luminosity,” ‘The Zoologist,’ July 15th, 1897, vol. i, No. 7, p. 304.
1903. HAUPT, H.—“Leuchtende Organismen,” ‘Naturwiss. Wochenschr.,’ 1903, p. 65.
1910. MANGOLD.—“Die Produktion von Licht,” ‘Handbuch der vergleichenden Physiologie.’
1914. FRIEND, H.—“Annelid Luminosity in Nottingham,” ‘Tr. Nat. Soc.,’ 1912–13 (1914).
1916. DAHLGREN, U.—“The Production of Light by Animals,” ‘Journ. Franklin Inst.,’ vol. 181, No. 5.

EXPLANATION OF PLATE XXIV.

FIG.

1. Light produced from earthworm placed on photographic plate.
2. Light produced from earthworm held stationary between two photographic plates.
3. Light produced from earthworm compressed between two photographic plates.



FIG. 1.

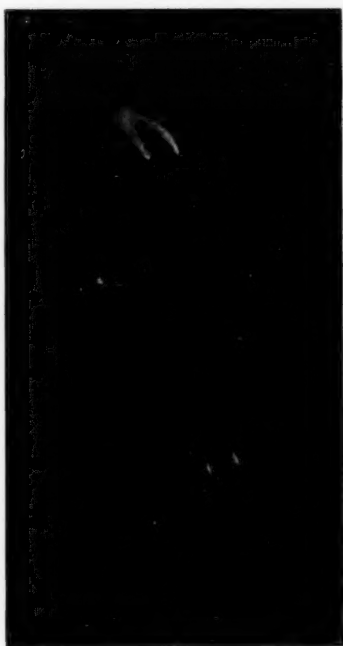


FIG. 2.



FIG. 3.